Weak attractor for the Klein-Gordon equation with a nonlinear oscillator in discrete space-time

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We consider the Klein-Gordon equation in the discrete space-time interacting with a nonlinear oscillator. In [1], we prove that the weak attractor of all finite energy solutions coincides with the set of all multifrequency solitary waves,

$$\sum_{j=1}^{N} \phi_j(x) e^{-i\omega_j t}, \qquad (x,\, t) \in \mathbb{Z}^n \times \mathbb{Z}, \quad \phi_j \in l^2(\mathbb{Z}^n), \quad \omega_j \in \mathbb{R} \mod 2\pi.$$

More precisely, we show that there are only one-, two-, and four-frequency solitary waves. In the continuous limit, only the one-frequency solitary waves survive. The convergence to the attractor takes place weakly (on finite subsets or in the weighted norms). The proof is based on a version of the Titchmarsh convolution theorem proved for distributions supported on a circle [3].

The result generalizes an earlier result for the Klein-Gordon equation in the continuous space-time [2].

References

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[3] Alexander I. Komech and Andrew A. Komech, *On the Titchmarsh convolution theorem for distributions on a circle*, Funktsional. Anal. i Prilozhen. **46** (2012), to appear (see arXiv:1108.2463).