

# Critical manifold in the space of contours in Stokes-Leibenson problem for Hele-Shaw flow

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We here deal with the Stokes-Leibenson problem for a punctual Hele-Shaw flow. By using a geometrical transformation inspired by Helmholtz-Kirchhoff method, we introduce an integro-differential problem which leads to the construction of a discrete model. We first give a short recall about the source-case: global in time existence and uniqueness result for an initial contour close to a circular one, investigation of the evolutionary structure of the solution. Our main subject concerns the development of a numerical model in order to get some qualitative properties of the motion. This model provides numerical experiments which confirm the existence of a critical manifold of codimension 1 in some space of contours. This manifold contains one attractive point in the source-case corresponding to a circular contour centered at the source-point. In the sink-case, every point of this manifold seems to be attractive. In particular, we present some numerical experiments linked to fingering effects.

## References

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