New phenomena in large systems of ODE and classical models of DC

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We consider the system

$$M\frac{d^2x_i}{dt^2} = -\frac{\partial U}{\partial x_i} + F(x_i) - A\frac{dx_i}{dt}, i = 1, ..., N$$

of N ordinary differential equations describing Newtonian dynamics of N particles (electrons), initially at the points

$$x_1(0) < x_2(0) < ... < x_N(0),$$

on the interval $[0, L) \in R$ with periodic boundary conditions, that is on the circle of length L. Here $M > 0, A \geq 0$ are the parameters, $F(x)$ is the external force, and

$$U(x_1, ..., x_N) = \sum_{i=1}^{N} \frac{\alpha}{|x_{i+1} - x_1|}.\alpha > 0,$$

(where of course $x_{N+1} \equiv x_1$) is the Coulomb repulsive interaction between nearest neighbors.

We review new results concerning this system: fixed points, quasi-homogeneous regime (Ohm's law) and very fast propagation of the "effective" external field, which is initially zero on the most part of the circle.

All these phenomena are closely related to many problems with DC (direct electric current), that the statistical physics was unable to understand. The following is a picturesque description of one of DC enigmas in the famous Feynman lectures, v. 6, pp. 33-34: "The force pushes the electrons along the wire. But why does this move the galvanometer, whis is so far from the force? Because when the electrons which feel the magnetic force try to move, they push - by electric repulsion - the electrons a little farther down the wire; they, in turn, repel the electrons a little farther on, and so on for a long distance. An amazing thing. It was so amazing to Gauss and Weber - who first built a galvanometer - that they tried to see how far the forces in the wire would go. They strung the wire all the way across the city."