Asymptotic solutions of the Navier-Stokes equations and scenario of turbulence development

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We discuss asymptotic solutions of the Navier-Stokes equations, describing periodic collections of vortices in 3D space. These solutions are connected with topological invariants of divergence-free vector fields. Equations, describing evolution of vortices, are defined on a graph – Reeb graph of the stream function or Fomenko molecule of the Liouville foliation. Homogenization with respect to the periodic structure leads to equations coinciding with Reynolds equations. It is well known that existence of the Reynolds stresses leads to the growth of the energy and entropy of the fluid. As the entropy reaches certain critical value, the molecules of the fluid have to form "clusters" which leads to the occurrence of turbulence.