On the general theory of multi-dimensional linear functional operators with applications in Analysis

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The talk is devoted to the linear multi-dimensional functional operator

$$(\mathcal{P}F)(x) := \sum_{j=1}^{N} c_j(x)(F \circ a_j)(x), \quad x \in D \subset \mathbb{R}^n.$$

Here $F \in C(I)$ with $I = (-1, 1)$ and $|F|$ norm in C, *coefficients* c_j and *arguments* a_j of $\mathcal{P}$ are continuous functions $D \to \mathbb{R}$ and $D \to I$, respectively; D is a domain with compact closure. These operators are of interest both in Analysis and in applying fields. If time allows I'll mention some problems in Integral geometry and PDE closely connected with them. As to the intrinsic problems relating to the operator $\mathcal{P}$ we'll discuss *the asymptotic behavior* of solutions to the equation $\mathcal{P}u = h_\varepsilon$ depending on a small parameter $\varepsilon \to 0$ under condition $h_\varepsilon = O(\varepsilon)$. Recent speaker's results make significally more precise analogous information based on the solution to well known Ulam problem. It turned out that this problem (as it is formulated in his book "A Collection of Mathematical Problems") is not well posed: the input information ($|\mathcal{P}F(x)| < \varepsilon$ for *all* $x \in D$) is redundant. As a matter of fact, to describe the asymptotic behavior of the function F the latter relation should be valid only at points x of some one-dimensional submanifold $\Gamma \subset D$ (subject to determining), but not everywhere in D. This result will be discussed together with a new Inverse problem for the equation $\mathcal{P}F = H_\varepsilon$ (reconstruction of the operator $\mathcal{P}$ using the given asymptotic behavior of the solution F).