A uniform Gronwall-type lemma with parameter and applications to nonlinear wave equations

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We discuss a uniform Gronwall-type lemma depending on a small parameter $\varepsilon > 0$, based on an integral inequality that predicts blow up in finite time of the involved unknown function for any fixed ε . The result permits to establish uniform estimates even if the function itself depends on ε .

As an application, we consider the asymptotics of the strongly damped nonlinear wave equation

$$u_{tt} - \Delta u_t - \Delta u + f(u_t) + g(u) = h$$

with Dirichlet boundary conditions, which serves as a model in the description of thermal evolution within the theory of type III heat conduction. In particular, the nonlinearity f acting on u_t is allowed to be nonmonotone and to exhibit a critical growth of polynomial order 5.

References

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