Eigenfunction of the Laplace operator in a tetrahedron

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Let T be an open and regular triangular pyramid (tetrahedron) in the space \mathbb{R}^3 with a boundary $\partial \Gamma$. Let α , β , γ , σ are barycentric coordinates of a point $(x, y, z) \in \mathbb{R}^3$ with respect to tetrahedron T which can be expressed in the variables x, y, z.

Theorem. The function $w = \sin(\alpha \pi/2) \sin(\beta \pi/2) \sin(\gamma \pi/2) \sin(\sigma \pi/2)$ is the eigenfunction of the Laplace operator $\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in *T*. The function *w* satisfies conditions: w > 0 in *T* and w = 0 on ∂T .

Let Π be unlimited cylinder in the space \mathbb{R}^4 which a cross-section with hyperplane is a quadrangular pyramid with edges of unit length (one-half of the octahedron). Let L be a second order linear differential operator in divergence form which uniformly elliptic with bounded measurable coefficients and η is its ellipticity constant. Let u be a solution of he mixed boundary value problem in Π for the equation Lu = 0 (u > 0) with homogeneous Dirichlet and Neumann data on the boundary of the cylinder. Our theorem allows us to continue this solution from the cylinder Π to the whole space \mathbb{R}^4 with the same ellipticity constant η .

This continuation allows us to prove a number of theorems about growth of the solution u in the cylinder Π .

The idea of using barycentric coordinates is taken from paper of A.P. Brodnikov, where it is used for the finding of eigenfunction of the Laplace operator in the triangle. Eigenfunction of the Laplace operator in hypertetrahedron from \mathbb{R}^4 and in n + 1-dimensional simplex from \mathbb{R}^n $(n \ge 2)$ were constructed by the author.