Classical solutions of the Vlasov–Poisson equations in a half-space

A.L. Skubachevskii
Peoples' Friendship University of Russia, Moscow 117198, Russia
skub@lector.ru

We consider the Vlasov system of equations describing the evolution of distribution functions of the density for the charged particles in a rarefied plasma. We study the Vlasov system in $\mathbb{R}^3_+ \times \mathbb{R}^3$ with initial conditions for distribution functions $f^\beta\big|_{t=0} = f_0^\beta(x,p)$, $\beta = \pm 1$, and the Dirichlet or Neumann boundary conditions for the potential of an electric field for $x_1 = 0$, where $f_0^\beta(x,p)$ is the initial distribution function (for positively charged ions if $\beta = +1$ and for electrons if $\beta = -1$) at the point x with impulse p, $\mathbb{R}^3_+ = \{x \in \mathbb{R}^3 \colon x_1 > 0\}$. Assume that initial distribution functions are sufficiently smooth and $\operatorname{supp} f_0^\beta \subset (\mathbb{R}^3_\delta \cap B_\lambda(0)) \times B_\rho(0)$, $\delta, \lambda, \rho > 0$, and the magnetic field $H(x)$ is also sufficiently smooth and has a special structure near the boundary $x_1 = 0$, where $\mathbb{R}^3_\delta = \{x \in \mathbb{R}^3 \colon x_1 > \delta\}$. Then we prove that for any $T > 0$ there is a unique classical solution of the Vlasov system in $\mathbb{R}^3_+ \times \mathbb{R}^3$ for $0 < t < T$ if $\|f_0^\beta\| < \varepsilon$, where $\varepsilon = \varepsilon(T, \delta, \rho, \|H\|)$ is sufficiently small.

This work was supported by the RFBR (grant No. 10-01-00395).