On the Gauss problem with Riesz potential

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This is a lecture on joint work with H. Harbrecht (U. Basel, Switzerland), G. Of (TU. Graz, Austria) and N. Zorii (Nat. Academy Sci. Kiev, Ukraine).

In $\mathbb{R}^n$, $n \geqslant 2$, we study the constructive and numerical solution of minimizing the energy relative to the Riesz kernel $|\mathbf{x} - \mathbf{y}|^{\alpha-n}$, where $1 < \alpha < n$, for the Gauss variational problem, considered for finitely many compact, mutually disjoint, boundaryless $(n-1)$–dimensional Lipschitz manifolds Γ_ℓ, $\ell \in L$, each Γ_ℓ being charged with Borel measures with the sign $\alpha_\ell = \pm 1$ prescribed. We show that the Gauss variational problem over an affine cone of Borel measures can alternatively be formulated as a minimum problem over an affine cone of surface distributions belonging to the Sobolev–Slobodetski space $H^{-\varepsilon/2}(\Gamma)$, where $\varepsilon := \alpha - 1$ and $\Gamma := \bigcup_{\ell \in L} \Gamma_\ell$. This allows the application of simple layer boundary integral operators on Γ and, hence, a penalty approximation. A corresponding numerical method is based on the Galerkin–Bubnov discretization with piecewise constant boundary elements. For $n = 3$ and $\alpha = 2$, multipole approximation and in the case $1 < \alpha < 3 = n$ wavelet matrix compression is applied to sparsify the system matrix. Numerical results are presented to illustrate the approach.

References

[1] G. Of, W.L. Wendland and N. Zorii: *On the numerical solution of minimal energy problems.* Complex Variables and Elliptic Equations **55** (2010) 991–1012.

[2] H. Harbrecht, W.L. Wendland and N. Zorii: *On Riesz minimal energy problems.* Preprint Series Stuttgart Research Centre for Simulation Technology (SRC Sim Tech) Issue No. 2010–80.