Infinite energy solutions for damped Navier–Stokes equations in $\mathbb{R}^2$

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The so-called damped Navier–Stokes equations in the whole 2D space:

$$\begin{cases} \partial_t u + (u, \nabla_x)u = \Delta_x u - \alpha u + \nabla_x p + g, \\ \operatorname{div} u = 0, \quad u\big|_{t=0} = u_0, \end{cases}$$

where α is a positive parameter, will be considered and the results on the global well-posedness, dissipativity and further regularity of weak solutions of this problem in the uniformly-local spaces $L^2_b(\mathbb{R}^2)$ will be presented. These results are obtained based on the further development of the weighted energy theory for the Navier–Stokes type problems. Note that any divergent free vector field $u_0 \in L^\infty(\mathbb{R}^2)$ is allowed and no assumptions on the spatial decay of solutions as $|x| \to \infty$ are posed.

In addition, the applications to the classical Navier–Stokes problem in $\mathbb{R}^2$ (which corresponds to $\alpha = 0$) will be also considered. In particular, the improved estimate on the possible growth rate of spatially non-decaying solutions as time goes to infinity:

$$\|u(t)\|_{L^2_b(\mathbb{R}^2)} \le C(t^5 + 1),$$

where C depends on u_0 and g, but is independent of t, will be presented. Note that the previous best known estimate was super-exponential in time:

$$\|u(t)\|_{L^\infty(\mathbb{R}^2)} \le C_1 e^{C_2 t^2},$$

see [1].

References

[1] O. Sawada and Y. Taniuchi, *A remark on L^∞-solutions to the 2D Navier–Stokes equations*, J. Math. Fluid Mech., **9** (2007), 533–542.