Wednesday math 17.2  Section 7.3
Continue

5.  7.2

\[ h \]
\[ \text{x distance from top} \]
\[ A(x) = (L_x)^2 \]

What is \( L_x \)?
\[ L_0 = a, \quad L_0 = b \]

In general
\[ L_x = a + x \left( \frac{b-a}{b} \right) \]

So
\[ u = \int_0^b \left( a + \frac{x}{b} (b-a) \right)^2 \, dx \]

7.3. Cylindrical Shells:

\[ \int_a^b 2\pi f(x) \, dx = \text{Vol} \]

area under graph
rotated about y-axis

but can also be adopted to rotate about x-axis

Ex. 3

\[ y = \sqrt{x} \quad \text{from } 0 \text{ to } 1 \]

about x-axis

\[ y = \sqrt{x} \land x = y^2 \]

\[ \int_0^1 2\pi y \sqrt{1-y^2} \, dy \]

for each cylinder,

height = \( 1-y^2 \), radius \( y \)

thickness \( dy \)
use of another axis - principle same

\[ y = x - x^2, \quad y = 0, \quad \text{about } x = 2 \]

cylinders parallel to \( x = 2 \)

\[ V \approx \sum_{i=1}^{n} 2\pi (2 - x_i^*) (x_i^* - x_{i-1}^*) \Delta x_i \]

as \( \| P \| \to 0 \)

get

\[ V = \int_0^1 2\pi (2 - x)(x - x^2) \, dx \]

\[ = \frac{\pi}{2} \]
7.4 Work, \( W = \int F \, dx \)  \\
\[ F = \text{Force}, \quad m = \text{mass}, \quad \Delta x = \text{distance} \]

in metric system  \( \text{mass} = \text{kg}, \text{dist} = \text{meters}, \text{time} = \text{sec} \)

newton = \( \text{kg} \cdot \text{m/s}^2 \)

If \( \text{const}, \text{acc.}, \) \( F = \text{const} \)

\[ W = F \cdot \Delta x \]

metric system: \( \text{newton} \cdot \text{meters} = \text{Joule} \)

english: \( \text{foot} \cdot \text{pound} = 1.36 \text{J} \)

ex 1: 1.2 kg book \( 0.7 \text{m} \) high?

\[ 0.7 \times 1.2 = 0.84 \text{J} \]

a) \( F = 1.2 \times 9.8 = 11.76 \text{N} \)

\( W = 0.7 \times 11.76 \text{N} = 8.2 \text{J} \)

b) \( 20\# \) 6 ft, \( F = 20\#, \ d = 6 \)

\( W = 120 \text{ ft} \cdot \text{lbs} \)

Now if \( \text{Force} \) is not constant,

\[ W = \int F(x) \, dx \]

\[ W = \int_a^b f(x) \, dx \]
Suppose 40 N req to go from 10 cm. to 15 cm.
How much work to stretch from 15 cm to 16 cm.

Solution: First determine k in Hooke's Law for this example.

\[ F = k \cdot \Delta x \]

\[ 40 = k \cdot (15-10) \]

So \( k = \frac{40}{5} = 8 \]

\[ W = \frac{1}{2} k \cdot \Delta x^2 \]

\[ W = \frac{1}{2} \cdot 800 \cdot 5^2 \]

\[ W = 400 \cdot 25 = 10000 \]

\[ = 39.4 \cdot 10^2 \cdot 10^{-4} = 156 \cdot 10^{-2} = 1.56 \text{ J} \]
ex 4,  

height 10 m  

base radius  

pump water to top  

of tank  

density water = \frac{1000 \text{ kg}}{m^3}  

\begin{align*}
\Gamma_i &= \frac{4}{10} \quad \Gamma_i = \frac{2}{5} \left(10 - x_i^*\right) \\
\end{align*}  

\text{vol. flow} = \pi r_i^2 \Delta x_i  

= \frac{4\pi}{25} \left(10 - x_i^*\right)^2 \Delta x_i  

m_i = \text{density \times volume}  

= 1000 \frac{4\pi}{25} \left(10 - x_i^*\right)^2 \Delta x_i = 1607 \pi \left(10 - x_i^*\right)^2 \Delta x_i  

\text{so } F_i = m_i g = (9.8) (160)(10 - x_i^*)^2 \Delta x_i  

= 1570 \pi (10 - x_i^*)^2 \Delta x_i  

\text{so } \omega_i = x_i^* F_i = 1570 \pi x_i^* (10 - x_i^*) \Delta x  

\sum \omega_i = \omega  

as \ \lim_{N \to \infty} \int_0^1 1570 \pi x (10 - x)^2 \, dx  

= 1570 \pi \cdot \frac{2048}{3} \approx 3.4 \times 10^8 \text{ C}