

CombinaTexas 2019

March 23–24, 2019

All the activities will be held at Blocker Building on the campus of Texas A&M University. The registration and breaks are at BLOC 141. All the plenary talks will be at the main lecture room, BLOC 166. For contributed talks, session A is in BLOC 166, and session B is in BLOC 164.

Saturday, Mar. 23, 2019

08:00–08:30: Registration and Breakfast (BLOC 141)
08:30–09:30: Plenary Talk 1: Joel Spencer (BLOC 166)
09:30–10:50: Contributed Session 1 (BLOC 166 & 164)
10:50–11:10: Break (BLOC 141)
11:10–12:10: Plenary Talk 2: Svetlana Poznanovikj (BLOC 166)

12:10–02:00: Lunch

02:00–03:00: Plenary Talk 3: Brendon Rhoades (BLOC 166)
03:00–04:20: Contributed Session II (BLOC 166 & 164)
04:20–04:40: Break (BLOC 141)
04:40–05:40: Plenary Talk 4: Criel Merino (BLOC 166)
06:00–08:00: Conference Dinner (catered) (BLOC 141)

Sunday, Mar.24, 2019

08:00–08:30: Breakfast (BLOC 141)
08:30–09:50: Contributed Session III (BLOC 166 & 164)
09:50–10:10: Break (BLOC 141)
10:10–11:10: Plenary Talk 5: Jessica McDonald (BLOC 166)
11:10–12:10: Plenary Talk 6: Ira Gessel (BLOC 166)

Schedule for Contributed Talks

Saturday Morning, Contributed Session I		
	Session A, BLOC 166	Session B, BLOC 164
09:30–09:50	Humberto Bautista Serrano	Suk Seo
09:50–10:10	Andrés Carnero	Christy Graves
10:10–10:30	Lucas Rusnak	Martin-Eduardo Frias-Armenta
10:30–10:50	Charles Burnette	Hector Alfredo Hernandez-Hdez
Saturday Afternoon, Contributed Session II		
	Session A, BLOC 166	Session B, BLOC 164
3:00–3:20	Kassie Archer	Rupei Xu,
3:20–3:40	Suho Oh	Ali Dogan
3:40–4:00	Somabha Mukherjee	JD Nir
4:00–4:20		Chun-Hung Liu
Sunday Morning, Contributed Session III		
	Session A, BLOC 166	Session B, BLOC 164
8:30–8:50	Ahmed Ashraf	Derek Drumm
8:50–9:10	Joshua Swanson	Caleb Ji
9:10–9:30	Tri Lai	Pani Seneviratne
9:30–9:50	Jacob White	Esmail Parsa

Abstracts–Plenary Talks

1. **Ira Gessel**, Brandeis University.

Title: *Rook theory and simplicial complexes*

Abstract: Rook theory deals with placements of nonattacking rooks on a board a subset of $[n] \times [n]$, where $[n] = \{1, 2, \dots, n\}$. The rook numbers of a board count placements of k nonattacking rooks on the board. The hit numbers of the board count placements of n nonattacking rooks on $[n] \times [n]$ in which k of the rooks lie on the board. In other words, the hit numbers count permutations π according to the number of pairs $(i, \pi(i))$ on the board. The fundamental identity of rook theory relates the rook numbers and hit numbers of a board.

The sets of nonattacking rook placement in $[n] \times [n]$ form a simplicial complex with the property that any two faces of the same size are covered by the same number of faces, and this property is all we need to prove the fundamental identity. Thus we can generalize the fundamental identity to other simplicial complexes with the same property. More generally, we can generalize it to simplicial posets. Interesting examples include matchings and trees of several types, including ordered and k -ary.

In this context we also have an analogue of the factorial rook polynomial of Goldman, Joichi, and White, and of its reciprocity theorem, which relates the rook numbers of a board to the rook numbers of a complementary board.

2. **Jessica McDonald**, Auburn University.

Title: *Packing and Covering Triangles in Graphs and Digraphs*

Abstract: In the 1980s, Tuza conjectured that that if a graph G has at most t pairwise edge-disjoint triangles, then there exists a set of at most $2t$ edges whose deletion makes the graph triangle-free. This conjecture is still wide open, and we will discuss some of what is known, including tight fractional approximations. We will also highlight new work (joint with Greg Puelo and Craig Tennenhouse) on a digraph analog of Tuza's Conjecture.

3. **Criel Merino**, UNAM, Mexico.

Title: *Some heterochromatic theorems for matroids*

Abstract: The anti-Ramsey number of Erdős, Simonovits and Sós from 1973 has become a classic invariant in Graph Theory. To extend this invariant to Matroid Theory, we use the heterochromatic number $hc(H)$ of a non-empty hypergraph H . The heterochromatic number of H is the smallest integer k such that for every colouring of the vertices of H with exactly k colours, there is a totally multicoloured hyperedge of H .

Given a matroid M , there are several hypergraphs over the ground set of M we can consider, for example, $C(M)$, whose hyperedges are the circuits of M , or $B(M)$, whose hyperedges are the bases of M . We determine $hc(C(M))$ for general matroids and characterise the matroids with the property that $hc(B(M))$ equals the rank of the matroid. We also consider the case when the hyperedges are the Hamiltonian circuits of the matroid. Finally, we extend the known result about the anti-Ramsey number of 3-cycles in complete graphs to the heterochromatic number of 3-circuits in projective geometries over finite fields, and we propose a problem very similar to the famous problem on the anti-Ramsey number of the p -cycles.

4. **Svetlana Poznanovikj**, Clemson University.

Title: *Properties of some combinatorial statistics: from permutations to words and labeled trees*

Abstract: Since the seminal result of MacMahon on the distribution of the major index over the symmetric group, several other Mahonian statistics have been found and studied together with partners such as left-to-right maxima, descents, excedences, etc. Later, Björner and Wachs defined a major index for labeled plane forests and showed that it has the same distribution as the number of inversions. This can be viewed as a generalization of MacMahon's result for permutations. In this talk I will discuss some of the classical permutation statistics in the setting of words and labeled forests. We will see what bottom-to-top maxima, cyclic bottom-to-top maxima, sorting index, and cycle minima are and show that the pairs (inv, Bt-max), (sor, Cyc), and (maj, Cbt-max) are equidistributed. Even though our results extend the result of Björner and Wachs and further generalize results for permutations, the picture is not complete and I will discuss some ideas on how to improve this.

5. **Brendon Rhoades**, University of California, San Diego.

Title: *Spanning subspace configurations*

Abstract: Let $k \leq n$ be positive integers. An ordered tuple of 1-dimensional subspaces (L_1, \dots, L_n) of a fixed k -dimensional vector space V is a *spanning line configuration* if $L_1 + \dots + L_n = V$ as vector spaces. I will discuss the geometry and combinatorics of these objects, generalizing classical results for the *flag variety* when $k = n$. I will also describe some (sometimes conjectural) extensions to higher-dimensional subspaces of V . Joint with Brendan Pawlowski and Andy Wilson.

6. **Joel Spencer**, Courant Institute (New York)

Title: *Four Discrepancies*

Abstract: Paul selects $\vec{c}_i \in \{-1, +1\}^n$ and Carole selects $\vec{x} = (x_1, \dots, x_n) \in \{-1, +1\}^n$. The payoff (which Carole tries to minimize) is $V = \max_i |\vec{x} \cdot \vec{c}_i|$.

When $\vec{c}_i \in \{0, 1\}^n$ we may interpret the matrix with columns \vec{c}_i as the incidence matrix of a family of sets and V is the discrepancy of the family.

We consider four variants of this problem Paul may play randomly (in which case Carole tries to minimize $E[V]$) or adversarially. Carole may play On-Line (selecting x_i after seeing \vec{c}_i) or Off-Line (seeing all \vec{c}_i and then selecting \vec{x}).

Abstracts–Contributed Talks

Saturday Morning, Contributed Session I

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	Session A, BLOC 166	Session B, BLOC 164
09:30–09:50	Humberto Bautista Serrano	Suk Seo
09:50–10:10	Andrés Carnero	Christy Graves
10:10–10:30	Lucas Rusnak	Martin-Eduardo Frias-Armenta
10:30–10:50	Charles Burnette	Hector Alfredo Hernandez-Hdez

[A. 9:30–9:50] Humberto Bautista Serrano, University of Texas at Tyler

Title: Intersection numbers and inconjugate intersection numbers for finite groups

Abstract. In a popular paper of Cohn, the concept of a covering number of a group was introduced. The covering number of a finite group G is the smallest number of proper subgroups of G whose set-theoretic union is G . Covering numbers are the subject of prior research by numerous authors, and in this talk we focus on a dual problem to that of covering numbers of groups, which involves maximal subgroups of finite groups. In addition, we will compare our results to some of the well-known results on covering numbers.

This is joint work by KASSIE ARCHER, HUMBERTO BAUTISTA SERRANO, KAYLA COOK, L.-K. LAUDERDALE, YANSY PEREZ, AND VINCENT VILLALOBOS

[A. 9:50–10:10] Andrés Carnero, UNAM

Title: Homology groups and total domination

Abstract: Let G be a simple graph. A set S of vertices is a total dominating set if each vertex in $V(G)$ is adjacent to some vertex in S . The cardinality of a smallest total dominating set in a graph G is called the total domination number of G and we will denote it by $\gamma_t(G)$. In this talk, we will present an upper bound on the total domination number of G by means of algebraic topology. The idea is to associate a simplicial complex $I(G)$, the independence complex, to G and give the bound in terms of its reduced homology groups. In 2002, Meshulam showed that, if $\tilde{H}_q(I(G)) \cong 0$, then $\gamma_t(G) \leq 2q + 2$. We will show an improvement to this bound for the cases $q = 1, 2$ when some additional restrictions to $\tilde{H}_q(I(G))$ are imposed.

[A. 10:10–10:30] Lucas Rusnak, Texas State University

Title: The category of incidence structures and generalizing graph theories

Abstract: An incidence structure is a combinatorial multi-design defined as a quintuple (V, E, I, f, g) of sets of vertices, edges, incidences, and a morphism pair (f, g) to assign incidences. Through the incidence structure, many graph theoretic theorems can be extended to integer matrices by examining locally graphic properties of the associated hypergraphs. We will survey some of my favorite hypergraphic generalizations before examining how closely the categorical structure of incidence hypergraphs are to well-studied combinatorial categories. We demonstrate the category of incidence hypergraphs is the natural category in which graph-like combinatorics resides and allows for a concrete and explicit construction of the quiver exponential.

[A. 10:50–11:10] Charles Burnette, Saint Louis University

Title: Permutations with equal orders

Abstract: Let $P(n)$ be the probability that two independent, uniformly random permutations of $[n]$ have the same order, and let $K(n)$ be the probability that they are in the same conjugacy class. Answering a question of Thibault Godin, we will see in this talk that $P(n) = n^{-2+o(1)}$ and that $\limsup \frac{P(n)}{K(n)} = \infty$. (This is based on joint work with Huseyin Acan, Sean Eberhard, Eric Schmutz, and James Thomas.)

[B. 9:30–9:50] Suk Seo, Middle Tennessee State University

Title: Fault-tolerant distinguishing sets in cubic graphs

Abstract: Assume a graph G models a facility with an intruder or a multiprocessor network with one malfunctioning processor. We want to use (the minimum possible number of) detectors to be able to precisely determine the location of the intruder or the malfunctioning processor. Various distinguishing set parameters have been defined based on the functionality of the detector such as locating dominating sets, identifying codes, and open-locating dominating sets. In this talk we consider several types of fault-tolerant detectors identified for the open-locating-dominating sets on cubic graphs.

[B. 9:50–10:10] Christy Graves, University of Texas at Tyler

Title: Uniformly most reliable 2-terminal networks

Abstract: Given a fixed amount of resources (vertices and edges), how should they be arranged to ensure that the network is most reliable? Mathematically, we define the two-terminal reliability polynomial of a graph with two specified target vertices to be the probability that there exists a path between the target vertices if each edge operates independently with the same fixed probability. Given a fixed number of vertices and a fixed number of edges, a graph is uniformly most reliable if its reliability polynomial is greater than or equal to all other graphs with the same number of vertices and edges for all probabilities between 0 and 1. We present specific cases for which a uniformly most reliable graph does not exist as well as cases where there does exist a uniformly most reliable graph.

[B. 10:10–10:30] Martin-Eduardo Frias-Armenta, Universidad de Sonora

Title: Contractible transformations of graphs and collapsibility

Abstract: In this talk we will give the definition of contractible graph given by Ivashchenko, we will see the definition of collapsible graph, we will prove that each contractible graph is collapsible and we will see different results around these concepts.

[B. 10:50–11:10] Hector Alfredo Hernandez-Hdez, Universidad de Sonora

Title: Programs to calculate Ivashchenko's and collapsible graphics.

Abstract: In this work, programs are presented for the calculation of a subfamily of Ivashchenko graphics and collapsible graphics as well as the required functions written in C / C ++ language.

A numbering of simple graphs is proposed and its own version is shown of the canonical labeling algorithm.

At the same time, it will be disseminated to repository of the Group of Geometric Structures and Combinatorics of the Universidad de Sonora.

Saturday Afternoon, Contributed Session II

Saturday Afternoon, Contributed Session II		
	Session A, BLOC 166	Session B, BLOC 164
3:00–3:20	Kassie Archer	Rupei Xu
3:20–3:40	Suho Oh	Ali Dogan
3:40–4:00	Somabha Mukherjee	JD Nir
4:00–4:20		Chun-Hung Liu

[A. 3:00–3:20] Kassie Archer, University of Texas at Tyler)

Title: Pattern avoidance and cycle type

Abstract: We say a permutation avoids a given pattern if there is no subsequence of the permutation that appears in the same relative order as the pattern. Pattern avoidance for permutations has been widely studied, but it remains open to enumerate certain sets of pattern-avoiding permutations with respect to their cycle type. For example, it is unknown how many cyclic permutations avoid any single pattern of length 3. In this talk, we discuss what is known and present a result for cycles avoiding certain pairs of permutations.

[A. 3:20–3:40] Suho Oh, Texas State University

Title: h-vector of some Gammoids

Abstract: H-vector comes from a simplicial complex by counting the faces of each dimension and shifting the sequence. Pure O-sequence comes from starting out with monomials of equal degree sum, collecting all their divisors and counting the monomials of each degree. Stanley has conjectured that h-vector of any matroid simplicial complex is a pure O-sequence as well. Gammoids come from describing the set of vertices that can be reached by vertex-disjoint paths in a directed graph (very different from usual graphic matroids!) By peeling off the layers of certain polytopes, we show that Stanley’s conjecture is true for a small class of gammoids including all cotrasversals.

[A. 3:40–4:00] Somabha Mukherjee, University of Pennsylvania

Title: Limiting Distribution of Quadratic Chaos on Graphs

Abstract: Given i.i.d. observations X_1, X_2, \dots, X_n from some distribution F_n , the quadratic chaos of F_n on a sequence of graphs G_n is defined as $T_n(F_n) := \sum_{(i,j)} a_{i,j}(G_n) X_i X_j$, where $a_{i,j}(G_n)$ is the $(i, j)^{th}$ element of the adjacency matrix of G_n . In this talk, we will provide sufficient conditions under which $T_n(F_n)$ converges weakly, assuming that the underlying distribution is Bernoulli with mean going to 0. The form of the limiting distribution is quite general, and covers many important examples as subcases. We will also demonstrate a universality phenomenon, that allows us to extend our result from the Bernoulli distribution to discrete distributions where $P(X_1 = 1)$ accounts for most of $E(X_1)$.

[B. 3:00–3:20] Rupei Xu, The University of Texas at Dallas

Title: Ulam Decompositions in Sparse Graphs

Abstract: Given two graphs G and H , each with n vertices and m edges, each graph edge set could be decomposed into r parts $E_G = E_{G_1} \cup E_{G_2} \cup \dots \cup E_{G_r}$ and $E_H = E_{H_1} \cup E_{H_2} \cup \dots \cup E_{H_r}$ such that G_i and H_i are isomorphic, this decomposition is called Ulam Decomposition and the minimum value of r is defined as $U(X, Y)$. Fan Chung, Ron Graham, Paul Erdős, Stan Ulam

and Frances Yao did a lot of contributions to prove the bounds of $U(X, Y)$ and generalized it to multiple pairs or even infinite pairs of graphs and hypergraphs.

Given two graphs G and H , the determination of whether $U(G, H) \leq k$ is an NP-complete problem, even when $k = 2$, it is still NP-complete. In this paper we apply first-order logic and structural graph theory tools to show the complexity result of Ulam decompositions in Sparse Graphs.

[B. 3:20–3:40] Ali Dogan, University of Houston Victoria

Title: On Saturated Graphs

Abstract: For a given graph H , we say that a graph G on n vertices is H -saturated if H is not a subgraph of G , but for any edge e in the complement of G the graph $G + e$ contains a subgraph isomorphic to H . The set of all possible values for the size of H -saturated graphs is called the edge spectrum for H -saturated graphs. In this talk, we will discuss the edge spectrum for H -saturated graphs when H is a path or a star. In particular, we investigate the second largest Path-saturated graphs. This is based on a joint work with Paul Balister.

[B. 3:40–4:00] JD Nir, University of Nebraska-Lincoln

Title: Turn-Type Questions about Cliques and Stars

Abstract: The classic extremal problem is that of computing the maximum number of edges in an F -free graph. In the case where F is a clique, the extremal number was determined by Turán. In 2015, Alon and Shikhelman generalized this problem, asking how many copies of T can be in a graph without a copy of F (which is equivalent to Turán's problem when $T = K_2$ and F is a large clique). We consider the permutations of this problem when T and F are cliques and stars.

[B. 4:00–4:20] Chun-Hung Liu, Texas A&M University

Title: Killing subgraphs of large minimum degree in H -minor free graphs randomly

Abstract: Fix a graph H and an integer d , we consider the threshold probability $p(n)$ such that a random subgraph of an H -minor-free n -vertex graph obtained by keeping each edge independently with probability $p(n)$ contains a subgraph of minimum degree at least d . Determining such threshold for all pairs (H, d) is expected to be difficult as it gives a constant-factor approximation for the maximum number of edges of H -minor-free n -vertex graphs, for any graph H . Joint with Wei, we determine such threshold asymptotically for a large set of pairs (H, d) by proving a structural theorem for H -minor-free graphs which generalizes a result of Ossona de Mendez, Oum and Wood.

Sunday Morning, Contributed Session III

Sunday Morning, Contributed Session III		
	Session A, BLOC 166	Session B, BLOC 164
8:30–8:50	Ahmed Ashraf	Derek Drumm
8:50–9:10	Joshua Swanson	Caleb Ji
9:10–9:30	Tri Lai	Pani Seneviratne
9:30–9:50	Jacob White	Esmail Parsa

[A. 8:30–8:50] Ahmed Ashraf, University of Western Ontario

Title: Tiling character polynomials

Abstract: The conjugacy classes of symmetric group S_n as well as its irreducible characters are indexed by integer partitions $\lambda \vdash n$. We introduce the class functions on S_n that count the number of certain tilings of Young diagrams. The counting interpretation gives a uniform expression of these class functions in the ring of character polynomials, as defined by Mur-naghan. A modern treatment of character polynomials is given in Orellana and Zabrocki. We prove a relation between these combinatorial class functions in the (virtual) character ring. From this relation, we were able to prove Goupil’s generating function identity, which can then be used to derive Rosas’ formula for Kronecker coefficients of hook shape partitions and two row partitions.

[A. 8:50–9:10] Joshua Swanson, University of California, San Diego

Title: Cyclotomic generating function asymptotics

Abstract: It is a remarkable fact that the complex roots of many combinatorial generating functions are each either a root of unity or zero. We call such polynomials *cyclotomic generating functions* and study the asymptotics of their coefficients by exploiting a beautiful formula for their cumulants. Examples include the major index on words or standard tableaux, rank for weight space bases in semisimple Lie algebras, and Hilbert series of total intersections in weighted projective space. We will discuss some of these examples in detail as time permits. Joint work with Sara Billey and Matjaž Konvalinka.

[A. 9:10–9:30] Tri Lai, University of Nebraska-Lincoln

Title: Factorization Theorems for Tiling Enumerations of Regions with Holes

Abstract: We investigate several new factorization theorems in the tiling enumeration. In particular, we show that the tiling number of a hexagon with an arbitrary number of triangular holes can always be written as a product of the tiling number of a similar hexagon with some triangular holes shuffled (the orientation of a triangular hole is changed from up-pointing to down-pointing, and vice versa) and tiling numbers of several semi-hexagons. Intuitively, the shuffling of the triangular holes only changes the tiling number of the region by a simple factor. This result generalizes a number of known enumerations of regions with holes in the literature. It also implies a multi-parameter generalization of ‘dual’ of MacMahon’s theorem on plane partitions by Ciucu and Krattenthaler in the study of asymptotic tiling enumeration. Interestingly, similar factorizations also appear in many different types of regions. If time allows, q -analogues of the factorization theorems and their possible connections to Schur function identities are also discussed.

[A. 9:30–9:50] Jacob White, University of Texas Rio Grande Valley

Title: Universal Binomial Coefficients

Abstract: Lucasnomial coefficients and q -binomial coefficients are both generalizations of binomial coefficients that:

- 1) Satisfy a generalized Pascal recurrence,
- 2) Can be defined as ratios of generalized factorials, and
- 3) Can be computed as a weighted sum over lattice paths.

We introduce a new generalization of binomial coefficients, which we call universal binomial coefficients. We prove that any generalized binomial coefficient which satisfies 1) and 2) is a specialization of the universal binomial coefficients. We also give combinatorial interpretations for universal binomial coefficients.

[B. 8:30–8:50] Derek Drumm, Lamar University

Title: Constructing a Fulfilled NFL Schedule Using Design Theory Techniques

Abstract: The NFL provides exciting experience for the interested viewer. However, the process by which the football games are scheduled is not perfect. The construction of a sufficient NFL schedule must consider many possible restrictions, complicating the process of creation. This presentation will reinterpret the current NFL schedule restrictions in design and graph theoretic terms, provide a construction to build a schedule based on this reinterpretation, and then utilize the construction to build a possible NFL schedule for 2019.

[B. 8:50–9:10] Caleb Ji, Washington University in St. Louis

Title: Distinguishing numbers and generalizations

Abstract: The distinguishing number of a graph was introduced by Albertson and Collins as a measure of the amount of symmetry contained in the graph. Tymoczko extended this definition to faithful group actions on sets; taking the set to be the vertex set of a graph and the group to be the automorphism group of the graph allows one to recover the previous definition. In this talk, I will first illustrate some techniques for computing this number and apply them to answer some hitherto open questions. Then I will show how this concept can be extended to partitions, which produces a new partially ordered set on the partitions of a number. Finally I will introduce a symmetric function generalization of this notion and end with some open questions regarding it.

[B. 9:10–9:30] Pani Seneviratne, Texas A&M University-Commerce

Title: Circulant graphs and their codes

Abstract: In this talk we will explore linear codes obtained from circulant graphs. Connections between parameters of these codes and the defining set of the graph will be discussed.

[B. 9:30–9:50] Esmail Parsa, The University of Montana

Title: Aspects of Unique D -Colorability for Digraphs

Abstract: In this talk we show that the two definitions of uniquely D -colorable digraphs that are either in terms of automorphisms or by vertex partitions are not always equivalent, and study conditions under which they are equivalent. In response to the question that for what portion of digraphs the fore-mentioned conditions hold, using probabilistic method we prove that asymptotically almost surely every random digraph is a core for which these conditions do not hold.