

# MATH 302. Discrete Mathematics

## Extra-credit Assignment 1.

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Please show your argument and computation. Calculators and computers are not permitted.

1. Prove that  $\sqrt{2} + \sqrt{3}$  is irrational.
2. Write down the set  $\mathcal{P}(\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))))$ ? How many elements are there in this set?
3. Let  $x$  be a real number. Show that

$$\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor.$$

4. Prove that either  $2 \cdot 10^{500} + 15$  or  $2 \cdot 10^{500} + 16$  is not a perfect square.
5. After baking a pie for the two nieces and two nephews who are visiting her, Aunt Nellie leaves the pie on her kitchen table to cool. Then she drives to the mall to close her boutique for the day. Upon her return she finds that someone has eaten one-quarter of the pie. Since no one was in her house that day—except for the four visitors—Aunt Nellie questions each niece and nephew about who ate the piece of pie. The four “suspects” tell her the following:

Charles: Kelly ate the piece of pie.

Dawn: I did not eat the piece of pie.

Kelly: Tyler ate the pie.

Tyler: Kelly lied when she said I ate the pie.

If only one of these four statements is true and only one of the four committed this heinous crime, who is the vile culprit that Aunt Nellie will have to punish severely?