

# MATH 302 Discrete Mathematics

## Extra-credit Assignment 2.

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Please show your argument and computation. Calculators and computers are not permitted.

1. Let  $f_n$  be the  $n$ -th Fibonacci number. Prove that  $f_0f_1 + f_1f_2 + \cdots + f_{2n-1}f_{2n} = f_{2n}^2$  when  $n$  is a positive integer.
2. Find the solution to the recurrence relation

$$a_n = a_{n-1} + 2a_{n-2}$$

with  $a_0 = 2$  and  $a_1 = 7$ .

3. In how many ways can one choose 10 integers  $a_1, a_2, \dots, a_{10}$  from the range  $[1, 100]$  such that for any pair of the chosen numbers, the difference is at least 2?
4. Let  $(x_i, y_i)$ ,  $i = 1, 2, 3, 4, 5$  be a set of five distinct points with integer coordinates in the  $xy$  plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates.
5. For  $n \in \mathbb{Z}^+$ , consider the following sum

$$\sum_{i=1}^n \frac{1}{(2i-1)(2i+1)}.$$

make a conjecture about the formula for the exact value of this sum, and prove your conjecture.