

MATH 302 Discrete Mathematics

Extra-credit Assignment 2.

Please show your argument and computation. Calculators and computers are not permitted.

1. Let f_n be the n -th Fibonacci number. Prove that $f_0f_1 + f_1f_2 + \cdots + f_{2n-1}f_{2n} = f_{2n}^2$ when n is a positive integer.
2. Find the solution to the recurrence relation

$$a_n = a_{n-1} + 2a_{n-2}$$

with $a_0 = 2$ and $a_1 = 7$.

3. In how many ways can one choose 10 integers a_1, a_2, \dots, a_{10} from the range $[1, 100]$ such that for any pair of the chosen numbers, the difference is at least 2?
4. Let (x_i, y_i) , $i = 1, 2, 3, 4, 5$ be a set of five distinct points with integer coordinates in the xy plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates.
5. For $n \in \mathbb{Z}^+$, consider the following sum

$$\sum_{i=1}^n \frac{1}{(2i-1)(2i+1)}.$$

make a conjecture about the value of the sum, and prove your conjecture.