

MATH 302. Discrete Mathematics

Extra-credit Assignment 3.

Please show your argument and computation. Calculators and computers are not permitted.

1. How many positive perfect squares less than 10^6 are multiples of 24?
2. Prove that $\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}$.
3. How many ways are there to travel in xyz space from the origin $(0, 0, 0)$ to the point $(100, 200, 300)$ by taking steps one unit in the positive x direction, one unit in the positive y direction, or one unit in the positive z direction? (Moving in the negative x, y or z direction is prohibited, so that no backtracking is allowed.)
4. Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.
5. Let A be a set with 100 elements. How many relations on A that are reflexive and anti-symmetric?