Lecture on Rules of Inference

Discuss example: what do you know from the following facts?

1. X and Y both need 87 in final to reach 450 pts.
2. X got 92 in final.
3. Y didn’t get A.

Three basic rules

• **Modus Ponens** \((p, p \to q) \implies \therefore q\).
• **Modus Tollens** \((-q, p \to q) \implies \therefore -q\).
• **Hypothetical Syllogism** \((p \to q), (q \to r) \implies \therefore p \to r\).

Rule of inference: to show \(p \to q\) is true, only need to consider the case when \(p\) is true. Other easy rules:

• **Conjunction** \((p, q) \implies p \land q, (p \land q) \implies p\).
• **Disjunction** \(p \implies p \lor q, (p \lor q, -p) \implies \therefore q\).
• **Resolution** \((p \lor q, -p \lor r) \implies \therefore q \lor r\).

Also any logical equivalence

**Example 1.** *Build an argument for* \((p, p \to -q, -q \to -r) \implies \therefore -r*.

**Example 2.** *Build an argument for* \(p \to r, r \to s, t \lor -s, -t \lor u, -u\) *then* \(-p*.

A trick: Show the logic equivalence \(p \to (q \to r) \equiv (p \land q) \to r\).

To show that \(p \implies \therefore (q \to r)\), we can do \(p, q \implies \therefore r\).

**Example 3.** *Build an argument for* \(u \to r, (r \land s) \to (p \lor t), q \to (u \land s)\) *and* \(-t\) *imply* \(q \to p*.

How about \(p, p \lor q, q \to (r \to s)\) and \(t \to r\) imply \(-s \to -t\)?

– How to show “invalid”?