Solution for Extra 1.

1. Prove $\sqrt{3}$ is irrational.

Pf: By contradiction. Assume $\sqrt{3}$ is rational. Then $\sqrt{3}=\frac{p}{q}$, where $p, q \in \mathbb{Z}$, $q \neq 0$, and p, q relative prime.

But then $\sqrt{3}\, q = p \Rightarrow 3q^2 = p^2$

$\Rightarrow p$ is a multiple of 3. So $p = 3k$ and

$3q^2 = (3k)^2 \Rightarrow q^2 = 3k^2 \Rightarrow 3 \mid q$

Thus 3 is a common factor of p and q. Contradiction!

2. Prove $\sqrt{2}+\sqrt{3}$ is irrational.

Pf. By contradiction. If $\sqrt{2}+\sqrt{3} \in \mathbb{Q}$, so is $(\sqrt{2}+\sqrt{3})^2$.

That is, $5+2\sqrt{6} \in \mathbb{Q} \Rightarrow \sqrt{6} \in \mathbb{Q}$.

$\Rightarrow \exists\, p, q \in \mathbb{Z}$, $q \neq 0$, $(p, q)=1$, and $\sqrt{6}=\frac{p}{q}$

$\Rightarrow 6q^2 = p^2$. In particular, p^2 is even. So p is even

Substituting in $p=2k$, we get $6q^2 = 4k^2$

$\Rightarrow 3q^2 = 2k^2$.

Thus q^2 is even $\Rightarrow q$ is even.

This contradicts the assumption that $(p, q)=1$.

3. If $|S|=n$, then $|P(S)| = 2^n$.

$|\emptyset| = 0 \Rightarrow |P(\emptyset)| = 2^0 = 1$.

$\Rightarrow |P(P(\emptyset))| = 2^1 = 2$

$\Rightarrow |P(P(P(\emptyset)))| = 2^2 = 4$

$\Rightarrow |P(P(P(P(\emptyset))))| = 2^4 = 16$.

4. let $x = n + \varepsilon$ where $0 \le \varepsilon < 1$.

Case 1. $0 \le \varepsilon < \frac{1}{3}$. $\lfloor 3x \rfloor = 3n + \lfloor 3\varepsilon \rfloor = 3n$

$$\left.\begin{array}{l} \lfloor x \rfloor = n, \\ \lfloor x+\frac{1}{3} \rfloor = n \\ \lfloor x+\frac{2}{3} \rfloor = n \end{array}\right\} \Rightarrow \lfloor x \rfloor + \lfloor x+\tfrac{1}{3} \rfloor + \lfloor x+\tfrac{2}{3} \rfloor = 3n$$

LHS = RHS

Case 2. $\frac{1}{3} \le \varepsilon < \frac{2}{3}$. then $1 \le 3\varepsilon < 2$.

So $\lfloor 3x \rfloor = 3n + \lfloor 3\varepsilon \rfloor = 3n+1$.

$\lfloor x \rfloor = \lfloor x + \frac{1}{3} \rfloor = n$. $\lfloor x + \frac{2}{3} \rfloor = n+1$

$\Rightarrow$ $\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor = 3n+1$

Case 3. $\frac{2}{3} \le \varepsilon < 1$. then $2 \le 3\varepsilon < 3$

So $\lfloor 3x \rfloor = 3n + \lfloor 3\varepsilon \rfloor = 3n+2$

And $\lfloor x \rfloor = n$

$\lfloor x + \frac{1}{3} \rfloor = \lfloor x + \frac{2}{3} \rfloor = n+1$

$\Rightarrow$ $\lfloor x \rfloor + \lfloor x + \frac{1}{3} \rfloor + \lfloor x + \frac{2}{3} \rfloor = 3n+2 = \lfloor 3x \rfloor$.

5. This is a truth table problem. The primary cases are. "Who ate the pie."

Primary	Charles	Dawn	Kelly	Tyler
Charles ate the pie	F	T	F	T
Dawn	F	F	F	T
Kelly ---	T	T	F	T
Tyler .-	F	T	T	F

Only the 2nd row has exactly one true statement. So it is Dawn who ate the pie.