

**MATH 431. Structures and Methods in
Combinatorics
Extra-credit Assignment 2
Due on Thursday, April 3, 2008**

Please show your argument and computation. Calculators and computers are not permitted.

1. For each positive integer p , let $b(p)$ denote the unique positive integer k such that $|k - \sqrt{p}| < \frac{1}{2}$. For example, $b(6) = 2$ and $b(23) = 5$. Find the value of S where

$$S = \sum_{p=1}^{2008} b(p).$$

2. A frog is placed at the origin on the number line, and moves according to the following rule: in a given move, the frog advances to either the closet point with a greater integer coordinate that is a multiple of 3, or to the closet point with a greater integer coordinate that is a multiple of 13. A *move sequence* is a sequence of coordinates which correspond to valid moves, beginning with 0, and ending with 39. For example, 0, 3, 6, 13, 15, 26, 39 is a move sequence. How many move sequences are possible for the frog?
3. Show that a sequence of $mn + 1$ real numbers either contains an increasing subsequence of $m + 1$ numbers, or a decreasing subsequence of $n + 1$ numbers.
4. Let H_n be the number of ways of seating n married couples around a circular table so that no man is next to his wife. Show that

$$H_n = \sum_{k=0}^n (-1)^k \binom{n}{k} 2^k (2n - k - 1)!.$$

5. The increasing geometric sequence x_0, x_1, x_2, \dots consists entirely of integral powers of 3. Given that

$$\sum_{n=0}^7 \log_3(x_n) = 308 \quad \text{and} \quad 56 \leq \log_3 \left(\sum_{n=0}^7 x_n \right) \leq 57,$$

find the sequence.