

**MATH 431-200/500. Structures and Methods in  
Combinatorics  
Extra-credit Assignment 3.  
Due on Wednesday, May 7th, 2008**

---

Please show your argument and computation. Calculators and computers are not permitted.

1. Show that a simple graph (no loops or multiple edges) with  $n$  vertices is connected if it has more than  $(n - 1)(n - 2)/2$  edges.
2. Give a combinatorial proof that

$$\sum_{k=1}^n k \binom{n}{k}^2 = n \binom{2n-1}{n-1}.$$

Hints: Count in two ways the number of ways to select a committee, with  $n$  members from  $n$  mathematicians and  $n$  computer scientists, such that the chairperson is a mathematician.

3. How many binary strings of length  $n$ , where  $n \geq 4$ , contains exactly two occurrences of 01?
4. Consider an  $n$ -by- $n$  board in which there is a nonnegative number  $a_{ij}$  in the square in row  $i$  and column  $j$ , ( $1 \leq i, j \leq n$ ). Assume that the sum of the numbers in each row and in each column equals 1. Prove that it is possible to place  $n$  non-attacking rooks on the board at positions occupied by positive numbers.
5. An integer is called *parity-monotonic* if its decimal representation  $a_1 a_2 \dots a_k$  satisfies  $a_i < a_{i+1}$  if  $a_i$  is odd, and  $a_i > a_{i+1}$  if  $a_i$  is even. How many four-digit parity-monotonic integers are there?