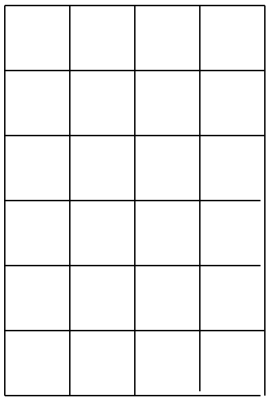
MATH 431-200/500. Structures and Methods in Combinatorics
Extra-credit Assignment 1. Due on Feb. 21, 2008

Please show your argument and computation. Calculators and computers are not permitted.

1. Each of 17 students talked with every other student. They all talked about three different topics. Each pair of students talked about one topic. Prove that there are three students that talked about the same topic among themselves.

2. In the 6×4 grid shown, 12 of the 24 squares are to be shaded so that there are two shaded squares in each row and three shaded squares in each column. Let N be the number of shadings with this property. Find the value of N.



3. A set S contains of 7 distinct positive integers, none is larger than 24. Show that there are two subsets of S with the same sum.

4. Let $(x)_k = x(x-1)(x-2)\cdots(x-k+1)$.

   Let x and y be real numbers and n a positive integer. Prove that

$$(x+y)_n = \sum_{k=0}^{n} \binom{n}{k} (x)_k (y)_{n-k}.$$

5. Using combinatorial argument to prove that for positive integers n and k,

$$\sum_{s=k}^{n} \binom{s}{k} \binom{n}{s} = 2^{n-k} \binom{n}{k}.$$