

MATH 630–600. Combinatorics

Fall 2008, Project Assignment

Imagine that you are asked to deliver a one-hour talk on a topic of combinatorics to hundreds of freshmen at Texas A&M! The project is to write down a short survey paper that you can hand out in the lecture hall right before your talk.

First, you need to decide on a topic. Any topic on enumerative combinatorics is fine. A list of suggestions is given at the end of this file. You can either choose one topic from the list, or write on your favorite topic. In the latter case, please discuss it with me before **October 15** to make sure that your topic is within the scope of this course.

Requirement.

1. Your paper should include a title, an abstract, several sections including an introduction, and a list of references. There should be at least five references cited.
2. Each topic in the suggested list is quite broad. It is impossible to cover everything of a topic. Please just focus on two to three aspects.
3. It is a survey paper, so there is no need to give detailed proofs for every result you state. On the other hand, you should state the definitions clearly.
4. The paper should contain at least five pages, but no more than 10 pages, (excluding the references).
5. Please use LaTeX to prepare your paper. Any version of TeX is fine.

Deadline. The project is assigned at the beginning of September. The deadline is **December, 1, Monday, in class**. No extensions, please.

Have fun and enjoy combinatorics!

Suggested Topics.

1. **Permutation statistics** A permutation statistic is a function from the symmetric group \mathcal{S}_n to the set of natural numbers \mathbb{N} . For example, the number of cycles, descent, excedance, inversion index, major index are all permutation statistics. Two famous families of permutation statistics are the Eulerian and MacMahonian statistics. Their distribution, as well as the joint distribution with other statistics, are important in understanding the combinatorial structure of many discrete objects.

2. **Pattern avoiding permutations.** One important and often difficult problem in the study of restricted permutations is the enumeration problem: Given a set R of permutations, enumerate the set $S_n(R)$ consisting of those permutations in S_n which avoid every element of R . The earliest solution to an instance of this problem seems to be MacMahon's enumeration of $S_n(123)$. Knuth shows that for any $\sigma \in S_3$, we have $|S_n(\sigma)| = C_n = \frac{1}{n+1} \binom{2n}{n}$, the n th Catalan number. In the last twenty years, the research on pattern avoiding permutations has blossomed, with hundreds of papers appeared every year.
3. **Continued fractions in Combinatorics.** Continued fraction is an analytic tool that is often used in number theory. But it is also equivalent to the generating function of weighted paths in the plane. This equivalence leads to continued fraction expansions for series involving known combinatorial quantities, including the Catalan numbers, the Bell and Stirling numbers, the tangent and secant numbers, the Euler and Eulerian numbers, etc. In many cases an explicit generating function can be derived from the continued fraction expansion.
4. **Combinatorial identities.** Combinatorial identities are identities between formulas arisen from enumerating combinatorial objects, which often involve binomial coefficients, Stirling numbers, Catalan numbers, Fibonacci numbers, etc. The subject encompasses all identities phrased in terms of recognized combinatorial entities, such as permutations, combinations, partitions, trees, graphs, and designs, and extends to other mathematical areas, for example, number theory, representation theory, algebra, geometry, and mathematics physics.
5. **Self-avoiding random walks.** A self-avoiding walk (SAW) is a path from one point to another which never intersects itself. Such paths are usually considered to occur on lattices, so that steps are only allowed in a discrete number of directions and of certain lengths. One important example is the self-avoiding walks on the Euclidean plane starting from the origin and stay within a restricted area. SAWs are often used to model the real-life behavior of chain-like entities such as solvents and polymers, whose physical volume prohibits multiple occupation of the same spatial point.
6. **Kernel method in combinatorics** The kernel method has recently become quite popular. Roughly speaking, in certain cases one obtains for a multivariate generating function a functional equation. For certain couplings of the variables, the denominator vanishes, but since one knows a priori that a power series expansion exists, one conclude that the numerator must also vanish. This is sufficient to compute the generating function, at least at special values, and subsequently in general.