MATH 630–600. Enumerative Combinatorics

Assignment 3. **Due on Wednesday, October 8, 2008**

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1. Verify the bijective proof given on Page 40 of Enumerative Combinatorics, between the set of partitions of n into odd parts, and the set of partitions of n into distinct parts, where n is a fixed positive integer.

2. Let $\binom{\mathbf{n}}{\mathbf{k}}_{\mathbf{q}}$ be the Gaussian coefficient. Prove the following identity:

$$\binom{\mathbf{n}}{\mathbf{k}}_{\mathbf{q}} = \mathbf{q}^{\mathbf{k}} \binom{\mathbf{n-1}}{\mathbf{k}}_{\mathbf{q}} + \binom{\mathbf{n-1}}{\mathbf{k-1}}_{\mathbf{q}}.$$

3. Let $p_k(n)$ be the number of partitions of integer n with exactly k parts. Compute the generating function

$$\sum_{n\geq 0} p_k(n) q^n.$$

4. Let $S(n,k)$ be the number of partitions of an n-set into k blocks, (i.e., the Stirling number of the second kind.) Prove that

$$\sum_{n\geq k} S(n,k) x^n = \frac{x^k}{(1-x)(1-2x)\cdots(1-kx)}.$$

5. Exercises on page 47–51.
Problems 20, 23(a, b, c, d), 42.