MATH 630–600. Enumerative Combinatorics

Assignment 6. **Due on Wednesday, November 19, 2008**

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1. Of the 16 four-element posets, (given in page 98), exactly one of them can not be built up from the poset **1** using the operations of disjoint union and ordinal sum. Find out that poset, and explain your reason.

2. Prove the following two lattice identities are equivalent.

$$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z), \tag{1}$$
$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z). \tag{2}$$

3. Let V be a vector space, and $L(V)$ be the lattice of subspaces of V. Not using the dimension, prove directly that $L(V)$ is modular. That is, for any three subspaces X, Y, Z of V with $X \subseteq Z$,
$$X \vee (Y \wedge Z) = (X \vee Y) \wedge (X \vee Z).$$

4. Let P be a finite partially ordered set, and let r be the largest size of a chain. Then P can be partitioned into r but no fewer antichains.

5. Let L be a finite lattice. Assume in L, if two elements x and y both cover $x \wedge y$, then $x \vee y$ covers both x and y. Prove that L is graded.

6. Let B_n be the Boolean algebra of rank nm where $n = 2k+1$. Assume that $\mathcal{A}$ is an anti-chain of size $\binom{n}{k}$. Let $\mathcal{A}_m$ be the sub-family of $\mathcal{A}$ which contains all subsets in $\mathcal{A}$ of size m. From Sperner Theorem, we know that $\mathcal{A}_m \neq \emptyset$ iff $m = k$ or $k+1$.
   Prove that $\mathcal{A}_k = \emptyset$, or $\mathcal{A}_{k+1} = \emptyset$.

7. Textbook, Exercise 27a on page 158.

8. Textbook, Exercise 31 on page 160.

9. Textbook, Exercise 44 on page 162.

10. Textbook. Exercise 45 on page 162.