CombinaTexas 2018
February 10–11, 2018

Saturday Morning, Contributed Session I

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	Session A, BLOC 166	Session B, BLOC 164
09:30–09:50	Yue Cai	Shaohui Wang
09:50–10:10	Ozlem Ugurlu	Lindsey-Kay Lauderdale
10:10–10:30	Xingwei Wang	Lucas Rusnak
10:30–10:50	Tri Lai	Randy Davila
10:50–11:10	Zachary Moring	Bhaswar Bhattacharya

[A. 9:30–9:50] Yue Cai, Texas A&M University

Title: *q-Stirling identities*

Abstract: In this talk we give combinatorial proofs of q-Stirling identities using restricted growth words. This includes a poset theoretic proof of Carlitz's identity, a new proof of the q-Frobenius identity of Garsia and Remmel and of Ehrenborg's Hankel q-Stirling determinantal identity. We also develop a two parameter generalization to unify identities of Mercier and include a symmetric function version.

[A. 9:50–10:10] Ozlem Ugurlu, Tulane University

Title: *Counting Borel Orbits in Symmetric Varieties of Types BI and CII*

Abstract: Let G be a reductive group, B be a Borel subgroup and let K be symmetric subgroup of G. The study of B orbits in G/K, or equivalently the study of K orbits in G/B has importance of Harish-Chandra modules and such a study comes with many interesting Schubert calculus problems. Although this subject is very well studied, it still has many open problems from combinatorial point of view. The most basic question that we want to be able to answer is that how many B orbits are there in G/K? In this talk, we will present our recent progress on this enumeration problem in the cases of classical symmetric varieties SO(2n+1, C)/S(O(2p, C) x O(2q+1, C)),

(Type BI), and Sp(2n, C)/(Sp(2p, C) x Sp(2q+1, C)), (Type CII). In particular, we will present our result on generating series of these numbers and we will show how our counting is related to the lattice path combinatorics.

[A. 10:10–10:30] Xingwei Wang, Nankai University and Texas A&M University

Title: *corners of (a,b)-core partitions*

Abstract: We introduce the concepts of stitches and anti-stitches, certain pairs of cells in a quotient space which we call wrap-up space. We prove that the anti-stitches of a rational Dyck path are in bijection with the segments of structure sets of the corresponding core partition, therefore the number of corners of a core partition can be counted by the number of stitches or anti-stitches. Based on these results, for coprime positive integers a and b, we give two essentially different formulae for the number of corners in all (a, b)-cores. This leads to an unexpected identity, expressing the rational Catalan numbers as weighed sums of binomial numbers. Moreover, we show that for an $(n, n+1)$-core partition λ determined by certain $(n, n+1)$-Dyck path P, the corners of λ correspond to pairs of consecutive right steps in P. As a consequence, we show that the number of $(n, n+1)$-cores with k corners is Narayana number $N(n, k+1)$. We also extend these results to multi-cores.

[A. 10:30–10:50] Tri Lai, University of Nebraska

Title: *Tiling Enumeration of Hexagons with Central Holes*

Abstract: Abstract: Motivated by MacMahon's classical theorem on plane partitions and by an open problem posed by Propp, we investigate lozenge tilings of a hexagon in which three chains of arbitrary numbers of triangles have been removed. It is the first time a region with both central holes and boundary holes has been considered in the field of enumeration of tilings. Our main theorem generalizes a number of known enumerations of hexagons with central holes, including Ciucu-Krattenthaler-Okada's work on 'punctured hexagons', Ciucu-Eisenkölbl-Krattenthaler-Zare's enumeration of 'cored hexagons', and Ciucu's recent result on 'F-cored hexagons'. Especially, our result implies two longstanding conjectures posed by Ciucu–Eisenkölbl–Krattenthaler–Zare as two very special cases.

[A. 10:50–11:10] Zachary Moring, Trinity University

Title: *A,B-Minimal Stirling Numbers*

Abstract: Let n be a positive integer, and denote by $P = B_1/B_2/\cdots/B_k \in \Pi_{n,k}$ a set partition of $I_n = \{1, 2, \ldots, n\}$ into k non-empty parts, where $\min(B_1) < \min(B_2) < \cdots < \min(B_k)$. A well-known result states that $|\Pi_{n,k}|$ is $S(n, k)$, the n, k-th Stirling number of the second kind. Several generalizations of $S(n, k)$ have appeared in the literature. We give a new generalization, the *A,B-minimal Stirling number of the second kind*; this new generalization not only subsumes many existing generalizations but also introduces some novel generalizations.

[B. 9:30–9:50] Shaohui Wang, Savannah State University

Title: *On the sharp lower bounds of Zagreb indices of graphs with given number of cut vertices*

Abstract: The first Zagreb index of a graph G is the sum of the square of every vertex degree, while the second Zagreb index is the sum of the product of vertex degrees of each edge over all edges. In our work, we solve an open question about Zagreb indices of graphs with given number of cut vertices. The sharp lower bounds are obtained for these indices of graphs in $\mathbb{V}_{n,k}$, where $\mathbb{V}_{n,k}$ denotes the set of all n-vertex graphs with k cut vertices and at least one cycle. As consequences, those graphs with the smallest Zagreb indices are characterized. (With Shengjin Ji)

[B. 9:50–10:10] Lindsey-Kay Lauderdale, University of Texas at Typer

Title: *Vertex-Minimal Planar Graphs with Cyclic Automorphism Groups*

Abstract: A theorem of Frucht states that for any finite group G, there exists a graph Γ such that the automorphism group of Γ is isomorphic G. Naturally, this result gave rise to numerous extremal problems in graph theory. For instance, vertex-minimal graphs with a prescribed automorphism group are the subject of prior research by numerous authors. In this talk, we will discuss vertex-minimal *planar* graphs with a cyclic automorphism group.

[B. 10:10–10:30] Lucas Rusnak, Texas State University

Title: *Oriented incidence and naturality in hypergraphic generalization*

Abstract: An oriented hypergraph provides for the natural generalization of graph theoretic concepts through its locally signed graphic sub-structure. In this talk, we will discuss how sub-monic path embeddings produce hypergraphic versions of Sachs Coefficient Theorem and Chaiken's All Minors Matrix-tree Theorem for hypergraphic adjacency and Laplacian matrices. The naturality of these theorems lies in the comparison of graph-like categories.

[B. 10:30–10:50] Randy Davila, University of Houston-Downtown

Title: *Conjecturing with TxGraffiti*

Abstract: Automated conjecturing seeks to write computer programs which formulate mathematical conjectures. Some well known examples include Faijtlowcz's Graffiti and DeLaVina's Graffiti.pc These conjecturing programs have led to a myriad of surprising results, and thus, have motivated the design and introduction of a new conjecturing program called TxGraffiti. Originally written by Davila during the summer of 2016, this program seeks to stimulate interest in graph theoretic parameters which have previously not been related. In this talk, we give a brief description of the program, as well as consider several conjectures of TxGraffiti on the zero forcing number of a graph. This represents the first known use of automated conjecturing to study the zero forcing number.

[B. 10:50–11:00] Bhaswar Bhattacharya, University of Pennsylvania

Title: *Birthday Paradox, Monochromatic Subgraphs, and the Second Moment Phenomenon*

Abstract: What is the chance that among a group of n friends, there are s friends all of whom have the same birthday? This is the celebrated birthday problem which can be formulated as the existence of a monochromatic s-clique K_s (s-matching birthdays) in the complete graph K_n, where every vertex of K_n is uniformly colored with 365 colors (corresponding to birthdays). More generally, for a general connected graph H, let $T(H, G_n)$ be the number of monochromatic copies of H in a uniformly random coloring of the vertices of the graph G_n with c colors.

In this talk we will discuss the limiting distribution of $T(H, G_n)$ in various asymptotic regimes. In particular, $T(H, G_n)$ exhibits an interesting second-moment phenomenon, converging to a Poisson distribution whenever its expectation and variance are asymptotically equal. As an application, we derive the distribution of $T(H, G_n)$, for Erdos-Renyi random graphs, where multiple phase-transitions emerge, depending on whether the graph H is balanced or unbalanced. We also discuss the asymptotics of $T(H, G_n)$, when G_n is a converging sequence of dense graphs (graphons).