

CombinaTexas 2018

February 10–11, 2018

Saturday Afternoon, Contributed Session II

February 10, Afternoon, Contributed Session II		
	Session A, BLOC 166	Session B, BLOC 164
2:55–3:15	Bo Lin	Jerry Hu
3:15–3:35	Federico Castillo	Stephen Graves
3:35–3:55	Li Ying	Archie Rowe
4:15–4:35	Anton Dochterman	Juan Alberto Rodrigues-Velazquez
4:35–4:55	Eric Bucher	Lohans de Oliveoira Miranda

[A. 2:55–3:15] Bo Lin, University of Texas at Austin

Title: *Tropical Fermat-Weber point*

Abstract: in a metric space, the Fermat-Weber points of a sample are statistics to measure the central tendency of the sample and it is well-known that the Fermat-Weber point of a sample is not necessarily unique in the metric space. We investigate the computation of Fermat-Weber points under the tropical metric on the quotient space $\mathbb{R}^n/\mathbb{R}\mathbf{1}$ with a fixed $n \in \mathbb{N}$, motivated by its application to the space of equidistant phylogenetic trees with N leaves (in this case $n = \binom{N}{2}$) realized as the tropical linear space of all ultrametrics. We show that the set of all tropical Fermat-Weber points of a finite sample is always a classical convex polytope, and we present a combinatorial formula for the minimal sum of distances from an arbitrary point to the points in the sample (which is attained at this set). We identify conditions under which this set is a singleton. We apply numerical experiments to analyze the set of the tropical Fermat-Weber points within a space of phylogenetic trees. We discuss the issues in the computation of the tropical Fermat-Weber points and the k -ellipses that are generalizations of the Fermat-Weber points. This is a joint work with Ruriko Yoshida.

[A. 3:15–3:35] Federico Castillo, Kansas University

Title: *Deformations of the nested braid fan*

Abstract: Generalized permutohedra are defined as polytopes whose normal fan coarsens the Braid fan. We wanted to generalize the construction so that we allowed a bigger set of edges. In joint work with Fu Liu, we introduce the nested Braid fan, which is a refinement of Braid by considering the first difference of ordered coordinates. We show that the nested Braid Fan is the normal fan of some polytope and study its combinatorics.

[A. 3:35–3:55] Li Ying, Texas A&M University

Title: *Stability of the Heisenberg product on symmetric functions.*

Abstract: The Heisenberg product is an associative product defined on symmetric functions which interpolates between the usual product and Kronecker product. I will give the definition of this product and describe some properties of it. One well known thing about the Kronecker product of Schur functions is the stability phenomenon discovered by Murnaghan in 1938. I will give an analogous result for the Heisenberg product of Schur functions.

[A. 4:15–4:35] Anton Dochtermann, Texas State University

Title: *Coparking functions and h-vectors of matroids*

Abstract: The h-vector of a simplicial complex X is a well-studied invariant with connections to commutative algebra and combinatorial topology. When X is the independence complex of a matroid Stanley has conjectured that its h-vector is a ‘pure O-sequence, i.e. the degree sequence of a monomial order ideal where all maximal elements have the same degree. The conjecture has inspired a good deal of research and is proven for some classes of matroids, but is open in general. Merino has established the conjecture for the case that X is a cographical matroid by relating the h-vector to properties of chip-firing and ‘G-parking functions’ on the underlying graph G . We introduce and study the notion of a coparking function on a graph (and more general matroids) inspired by a dual version of chip-firing. As an application, we establish Stanley’s conjecture for new classes of binary matroids that admit a well-behaved ‘circuit covering’. Joint work with Kolja Knauer.

[A. 4:35–4:55] Eric Bucher, Michigan State University

Title: *Component preserving mutations: building up maximal green sequences from sub-quivers.*

Abstract: Quiver mutation is a operation one can define on a directed graph that has shown to model the behavior of a large variety of mathematical objects. We will discuss a bit about quiver mutation, and then proceed to exploring quivers for a special sequence of mutations called maximal green sequences. The aim of the talk is to discuss recent work that allows one to build maximal green sequences for larger quivers by looking at “component preserving” sequences on induced subquivers. These new techniques have allowed us to construct maximal green sequences for large families of quivers where their existence was previously unknown.

[B. 2:55–3:15] Jerry Hu, University of Houston-Victoria

Title: *Values Sets of Polynomial Maps On Unequal Dimensional Spaces over Finite Fields*

Abstract: Let F_q be a finite field of cardinality q , and f be a polynomial map from F_q^n to F_q^n . Gary Mullen, Daqing Wan, Qiang Wang considered the problem of estimating the cardinality of the set of points missed by f , and give a lower bound of it if f is not surjective, relying on p -adic lifting of f . Michiel Kusters proved an improved lower bound using a different technique. We consider the problem in the case of unequal dimensional spaces: If f is a polynomial map from F_q^m to F_q^n , with m less than or equal to n . We want to estimate the cardinality of set of subvarieties of dimension $n - m$ and degree d missed by f . We prove a lower bound for the the cardinality of set of subspaces of dimension $n - m$ missed by f if there is such a subspace of dimension $n - m$.

[B. 3:15–3:35] Stephen Graves, The University of Texas at Tyler

Title: *Growth of Face-Homogeneous Tessellations*

Abstract: A tessellation of the plane is *face-homogeneous* if for some integer $k \geq 3$ there exists a cyclic sequence $\sigma = [p_0, p_1, \dots, p_{k-1}]$ of integers ≥ 3 such that, for every face f of the tessellation, the valences of the vertices incident with f are given by the terms of σ in either clockwise or counter-clockwise order. When a given cyclic sequence σ is realizable in this way, it may determine a unique tessellation (up to isomorphism), in which case σ is called *monomorphic*, or it may be the valence sequence of two or more non-isomorphic tessellations (*polymorphic*). A tessellation whose faces are uniformly bounded in the hyperbolic plane but not uniformly bounded in the Euclidean plane is called a *hyperbolic tessellation*. Such tessellations are well-known to have exponential growth. We seek the face-homogeneous hyperbolic tessellation(s) of slowest growth and show that the least growth rate of such monomorphic tessellations is the “golden mean,” $\gamma = (1 + \sqrt{5})/2$, attained by the sequences $[4, 6, 14]$ and $[3, 4, 7, 4]$. A polymorphic sequence may yield non-isomorphic tessellations with different growth rates. However, all such tessellations found thus far grow at rates greater than γ .

[B. 3:35–3:55] Archie Rowe, Tarleton University

Title: *Improving the bounds of Ramsey Numbers using Gibb’s Sampler and Deep Learning*

Abstract: The Ramsey number $R(m_1, \dots, m_k)$ is the minimum number of vertices n such that any k -coloring of the edges of the complete graph K_n , there is at least one color i for which there exists a clique of size m_i that is monochromatically i . These numbers are traditionally hard to establish analytically. They are also hard to find computationally because of the size of the space of possible colorings. We will discuss a search method which combines a Gibbs Sampler search algorithm and Deep Learning techniques. This allows us to move rapidly and intelligently through the state space, harnessing the power of modern NVIDIA GPU hardware and MCMC techniques. We will present our current partial progress toward increasing the lower bounds smaller, non-trivial Ramsey numbers.

[B. 4:15–4:35] **Juan Alberto Rodrigues-Velazquez, Universitat Rovira i Virgili**

Title: *On the k -metric dimension of metric spaces*

Abstract: The metric dimension of a general metric space was introduced in 1953 but attracted little attention until, about twenty years later, it was applied to the distances between vertices of a graph. Since then it has been frequently used in graph theory, biology, robotics and many other disciplines. The theory was developed further in 2013 for general metric spaces. More recently, the theory of metric dimension has been generalized, again in the context of graph theory, to the notion of a k -metric dimension, where k is any positive integer, and where the case $k = 1$ corresponds to the original theory. Here we develop the idea of the k -metric dimension both in graph theory and in metric spaces.

[B. 4:35–4:55] **Lohans de Oliveira Miranda, UFPI, UNISUL, Brazil**

Title: *Combinatorial Dickson's Method for Generating Pythagorean Triples*

Abstract: From the Newton's binomial formula and geometric procedures is established the Dickson's method for Pythagorean triples.