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Ancient Algebra¹

1 Origins.

Our word ‘Algebra’ is derived from the Arabic expression

al-jabr wa'l muqabala

which occurs in the title of the first Arabic text on algebra written by Al-Khwarizmi in the 9th century. We have the words:

Al jabr	—	restoration or completion (add equals to equals to eliminate negative terms)
Al-muqabala	—	reduction or balancing (cancelling terms that occur on both sides of an equation)

So, to Al-Khwarizmi, ‘Algebra’ is the *art of reducing and solving equations*. In modern algebra, the emphasis has shifted to structure. Its roots started with the work of Galois on the possibility of solving equations by means of radicals (1830).

2 Three Kinds of Algebra

In ancient times there were three kinds of algebra.

A. Mixed Algebra. This is Babylonian type in which line segments and areas, etc. are added together and set equal to numbers.

B. Numerical Algebra. Herein only rational numbers m/n are admitted as coefficients and solutions of equations. (cf. Diophantus)

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C. Geometric Algebra. Here line segments, areas, and volumes are kept strictly apart, e.g. when lengths are multiplied, area results. This is found in Greek mathematics, we know, but also in Chinese and Indian mathematics. Solutions to quadratics and linear equations are accomplished geometrically. In this connection, modern mathematicians such as Decartes, solved geometric problems by first converting them to algebraic ones and then the solutions back to geometric terms.