

Anaxagoras of Clazomenae¹

(c. 500- c 428 BC)

As with many mathematicians of the 5th century BCE, little is known of Anaxagoras' life. He was born in Clazomenae, now in Turkey. He was fully committed to science, and neglected his considerable wealth toward its end. Most probably, he fit within the Ionian school² as he like his predecessors was concerned with explaining phenomena in terms of matter or physical forces.

In about 480 he moved lived and thrived in Athens, enjoying the friendship of Pericles (c. 495 - 429 BCE, Athens). Pericles, an Athenian statesman largely responsible for the development, in the later 5th century BC, of Athenian democracy and the Athenian empire, making Athens the political and cultural focus of Greece. The arrival of the Sophist philosophers in Athens happened during his middle life, and he took full advantage of the society of Zeno. It is from the Sophists that Anaxagoras is said to have learned impassivity toward trouble and insult and skepticism toward divine phenomena. When Pericles suffered decline in influence, Anaxagoras lost influence as well, and was attacked for his "impious" views. In fact, the attack on Anaxagoras may well have been an indirect attack on Pericles. Imprisoned in about 450 BCE and then later released probably due to the intervention of Pericles, he was compelled to leave Athens to retire to Lampsacus for the remainder of his life. In the end, according to other sources he was condemned to death for advocating the Persian cause, at the ripe age of seventy-two.

The particular charge for his imprisonment was for asserting that the sun was not a diety but a huge red-hot stone as large as all of Peloponnesus and the moon borrowed its light from the sun. That is, moonlight is reflected light. This epoch making discovery permitted the first accurate description and explanation of solar and lunar eclipses. His full explanation was not without error, but the idea took root and became the accepted theory very soon thereafter.

Anaxagoras' theory of the cosmos was his most original contribution. He viewed that the cosmos was formed by mind in two stages: first, by a revolving and mixing process that still continues; and, second, by the development of living things. As to the first,

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²Thales, Anaximander, Anaximenes, Diogenes of Apollonia, et al.

he theorized that the formation of the world began with a vortex set up in a portion of mixed mass in which all things were together. This rotating motion began at the center and gradually spread in wider and wider circles. From this, the elements of the world began to separate into 'aether' and air. From the air was distilled the clouds, water, earth and stones. Using ideas of centrifugal force, he postulated the process of condensation of the air into the solid earth. These ideas were later used by Kant and Laplace to describe the formation of the solar system. Admittedly, these cosmological theories that are abstract, unverifiable, but vaguely plausible. This is how new science begins, with guesses and plausibility arguments long before experiments.

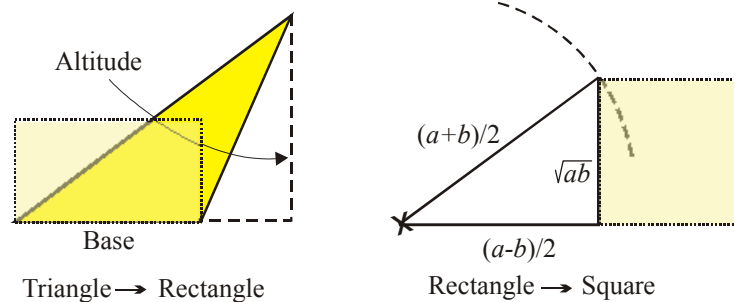
On the constitution of matter he differed from Greek thinkers of the time who had tried to explain the physical universe by an assumption of a single fundamental element (cf. Anaximander). Some, including Parmenides, asserted that such an assumption could not account for movement and change, and, others posited more elements (e.g. air earth, water, fire) as necessary for material explanation. Anaxagoras posited an infinite number. Unlike his predecessors Anaxagoras included materials found in living bodies, such as leaf, bark, flesh, and bone.

He wrote the book *On Nature*, the first widely circulated book on scientific subjects. Cost: 1 drachma.

Though Anaxagoras was mostly a natural philosopher rather than a mathematician, while in prison, he attempted to square the circle³ using **only** a straight edge and compass. More precisely, sources state that he "wrote" on this subject. The term "wrote" was at times used to indicate "studied" or "investigated". Note that it was probably known that regular polygons, of an arbitrarily large number of sides, inscribed in a circle could be squared using only the straight edge and compass. From this preliminary work, most likely, the general "squaring the circle" conjecture was made.

³One of the oldest problems in mathematics, the problem of squaring the circle was finally resolved in the negative by the German mathematician C. L. F. Lindemann (1852 - 1939) in about 1881. Precisely stated, this problem poses the question of constructing a square having the same area as a given circle. Though solutions were given by the ancients using a variety of methods, the goal was always to determine a solution using only a straight edge and compass. This problem has a remarkable history and has been one of those mathematical problems that attracted professional and amateur mathematicians alike. Even today, mathematics departments all over the world each year receive new straight edge and compass constructions. Of course all are incorrect.

Can you perform this construction? There are two steps. The first is to construct a rectangle having the same area as a triangle. For this use the triangle base with half the altitude. The second is to square the rectangle. To do this suppose the dimensions of the rectangle are a and b , with $a > b$. The area is ab , and the square with this area has side length \sqrt{ab} . Form a right angle and construct the length of one side to be $\frac{a+b}{2}$. From the endpoint construct an arc of radius $\frac{a-b}{2}$. Extend the second side of the right angle to meet this arc. By the Pythagorean theorem, the length of the second side is $\sqrt{(\frac{a+b}{2})^2 - (\frac{a-b}{2})^2} = \sqrt{ab}$. See the diagram below.



Now repeat the process for all the triangles constructed from the inscribed polygon (all with a vertex at the origin). Add them up and create the square.

The attempt conveys a remarkable amount of information. It shows clearly that that Greeks were more than just casually interested in non-practical problems, and that they had a very clear distinction between the exact and the approximate. It demonstrates a differentiation of acceptable and not acceptable methods, and this may have preceded or have developed along side acceptable logic for proofs. The price paid in this particular situation is that the Greeks were prepossessed by straight edge and compass constructions for much of their history. Thus, precluded from development was the vast range of functions known to us and the general concept of function, as well. In all of the ancient Greek literature, functions were always specific examples, whether the conic sections or trigonometric functions, or other dynamically formed curves.

Yet, Anaxagoras lived in the age where the great problems of antiquity were formed:

1. Doubling the cube. (Delian problem)

2. Trisecting the angle.
3. Squaring the circle.

Anaxagoras represents **bold, rational** inquiry. He represented the Greek trademark: “the desire to know”. His principle interest was in philosophy, where his main belief was that “reason rules the world.”

The Heroic Age

If Greek civilization seems familiar to the modern mind it is because the Greeks loved reason in both practice and form. It was a necessary aspect of cultural life, a necessary part of political persuasion, and a replacement for appeals to tradition and unchallenged religion. Science and theology, at times sister subjects, were simultaneously liberated together with political thought and became the principle parts of the unique adventure of the Greek mind. This period has been called the *heroic age*, for many reasons, but particularly for us because the Greeks attempted to solve these very difficult problems. That they were difficult is evidenced from that fact that it would be two millennia before their resolution was complete — all in the negative. The problems would tempt, perplex, and ultimately resist the efforts of the very best mathematicians of every age until the 19th century, with the circle squaring problem resolved in 1881. Attempts to solve these problems and other ancient problems would drive the developmental directions of mathematics until modern times.