

Greek Numbers and Arithmetic¹

1 Introduction

The earliest numerical notation used by the Greeks was the Attic system. It employed the vertical stroke for a one, and symbols for “5”, “10”, “100”, “1000”, and “10,000”. Though there was some streamlining of its use, these symbols were used in a similar way to the Egyptian system, being that symbols were used repeatedly as needed and the system was non positional. By the Alexandrian Age, the Greek **Attic** system of enumeration was being replaced by the Ionian or alphabetic numerals. This is the system we discuss.

The (Ionian) Greek system of enumeration was a little more sophisticated than the Egyptian though it was non-positional. Like the Attic and Egyptian systems it was also decimal. Its distinguishing feature is that it was alphabetical and required the use of more than 27 different symbols for numbers plus a couple of other symbols for meaning. This made the system somewhat cumbersome to use. However, calculation lends itself to a great deal of skill within almost any system, the Greek system being no exception.

2 Greek Enumeration and Basic Number Formation

First, we note that the number symbols were the same as the letters of the Greek alphabet.

symbol	value	symbol	value	symbol	value
α	1	ι	10	ρ	100
β	2	κ	20	σ	200
γ	3	λ	30	τ	300
δ	4	μ	40	υ	400
ϵ	5	ν	50	ϕ	500
ζ	6	ξ	60	χ	600
η	7	\omicron	70	ψ	700
θ	8	π	80	ω	800
	9	φ	90	$\gamma\lambda$	900

where three additional characters, the ς (digamma), the φ (koppa),

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and the λ (sampi) are used. Hence,

$$\sigma\pi\zeta = 287$$

Larger Numbers

Larger numbers were also available. The thousands, 1000 to 9000, were represented by placing *diacritical* mark ' before a unit. Thus

$$'\gamma\sigma\pi\zeta = 3287$$

In other sources we see the diacritical mark placed as a subscript before the unit. Thus

$$,\gamma\sigma\pi\zeta = 3287$$

The uses of a M was used to represent numbers from 10,000 on up. Thus

$$M^\epsilon = 50,000 \quad M^{\alpha\kappa}\sigma\pi\zeta = 120,287$$

Alternatively, depending on the history one reads

$$M\epsilon = 50,000 \quad M\alpha\kappa \cdot \sigma\pi\zeta = 120,287$$

Archimedes, in his book *The Sand Reckoner*, calculated the number of grains of sand to fill the universe². This required him to develop an extension the power of Greek enumeration to include very large numbers.

Fractions

In the area of fractions, context was crucial for correctly reading a fraction. A *diacritical* mark was placed after the denominator of the (unit) fraction. So,

$$\beta' = \frac{1}{2} \quad \text{and} \quad \mu\beta' = \frac{1}{42}$$

but this latter example could also mean $40\frac{1}{2}$.

More complex fractions could be written as well, with context again being important. The numerator was written with an overbar. Thus,

$$\overline{\nu\alpha} \pi\delta = \frac{51}{84}$$

Numerous, similar, representations also have been used, with increasing sophistication over time. Indeed, Diophantus uses a fractional form identical to ours but with the numerator and denominator in reversed positions.

²The reader may ask, "What universe?" It was the universe of Aristarchus, the so called 'ancient Copernicus' because he proposed a sun centered universe with the earth and other planet orbiting it. More on this later.

3 Calculation

The arithmetic operations are complex in that so many symbols are used. However, as you can imagine, addition amounts to grouping and then carrying. For example $5 + 7 = \epsilon + \zeta = \kappa\beta = 12$, not terribly unlike what we do. Multiplication was carried out using the distributive law. For example:

$$\begin{aligned}\sigma\pi\zeta \cdot \beta &= (\sigma + \pi + \zeta) \times \beta \\ &= (200 + 80 + 7) \times 2 \\ &= 400 + 160 + 14 = 574 \\ &= \phi\omicron\delta\end{aligned}$$

Remarkably, division was performed in essentially the same way as we do it today.