Algebra in the Renaissance

Problems

1. Apply the Cardano algorithm to solve the cubic equation $x^3 = 8x + 3$, which has the obvious solution $x = 3$. Can you obtain the solution?

2. Apply the Bombelli *ansatz* to resolve the answer to the above question.

3. (i) Show that trisecting the angle 60º is equivalent to solving the cubic $y^3 - 3y = 1$. (ii) One important step in showing that the angle 60º cannot be trisected is to show that the cubic $y^3 - 3y = 1$ has no rational solutions. Show this. (Hint. Begin with the assumption that $y = \frac{p}{q}$ is a rational solution where p and q are relatively prime. Derive a contradiction. Alternaatively, apply the Cardano formula.)

4. Show the essential step of Bombelli's argument, if $2 + ib = \sqrt[3]{2 + 11i}$ this implies that $b = 1$.

5. Generalize your conclusion in the problem above to show that $y^3 - ay = 1$ has no rational solutions for positive a.

6. Complete Mazzinhi's solution to the problem, " Find two numbers such that multiplying one by the other makes 8 and the sum of their squares is 27," using his tranformations. Solve the problem directly by first completing the square.

7. Derive alà Kepler the volume of a sphere to be $V = \frac{1}{3}r\,S$, where r is the radius and S is the surface area. Make your division of the sphere into solid angles and use the formula for the volume of a pyramid. ($V_{\text{pyramid}} = \frac{1}{3}hb^2$, where h is the height and b is the side length of the base.)

8. What steps must be taken to make the argument above rigorous?