

Calculus

Problems

1. Do you sense a change in basic philosophy in the mathematicians of this period. What are the ties to the ancient world? What are the dogmas that restrict or promote the development of mathematics? How do the mathematicians of this time compare with contemporary mathematicians, except for the wide chasm of knowledge?
2. Prove Vieta's formula for $\cos nx$.
3. Fill in the details of the derivation of Vieta's formula for π .
4. Using Maple, if you have it, or a calculator, determine the first five approximations of π using this formula. (From the formula

$$\frac{\pi}{2} = \frac{1}{\sqrt{\frac{1}{2}}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}} \cdots}$$

The first approximation is the reciprocal of

$$p_1 = \sqrt{\frac{1}{2}}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}$$

The second approximation is the reciprocal of

$$p_2 = \sqrt{\frac{1}{2}}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}}$$

and so on. It might be better expressed as a recursion relation as described (as an algorithm) in the text.

5. Describe (with historical dates of origin) the fundamental principles that can be used to compute π . Illustrate them with an example.
6. Making a logarithm table: Show how knowing $10^{\frac{1}{n}}$ for $n = 1, 2, \dots, 30$ can be used to build a table of (common) logarithms. (This is what Briggs accomplished.)
7. Attempt to show that $\int_0^a x^2 dx = \frac{a^3}{3}$, using Cavalieri's method of indivisibles. (This may prove difficult.)

8. Prove the theorem of Fermat: **Theorem.** Given any number of fixed lines, the locus of a point such that the sum of the squares of the segments drawn at given right angles from the point to the lines is constant, is a solid locus (ellipse).
9. Prove that the largest cylinder that can be inscribed in a sphere is one in which the ratio of diameter to altitude is $\sqrt{2} : 1$.