

Math 325, Quiz 8A
October 31, 2008

PRINT NAME _____

Show enough work to support your answers, clearly indicate your answers. All rates are continuously compounded.

ONE (2 pts) If the forward rate is at most 5% at all times $0 \leq t \leq 2$, prove that the two year spot rate is at most 5%.

SOLUTION: If $f(t) \leq .05$ for all $0 \leq t \leq 2$ then $s(2) = \frac{1}{2} \int_0^2 f(u) du \leq \frac{1}{2} \int_0^2 0.05 du = .05$

TWO (1 pt @) Is the statement TRUE or FALSE about every short rate model? (Circle your choice)

- (a) **TRUE** FALSE $P(T, T, r_T) = 1$ for all values of $T \geq 0$
- (b) TRUE **FALSE** The value of $P(1, 3, 0.08)$ must be $\exp(-0.16)$.
- (c) TRUE **FALSE** Each price $\$P(0, T, r_T)$, $T \geq 0$, is the current market prices of a par \$1 zero maturing in T years.
- (c) **TRUE** FALSE The numbers $\delta(T) = P(t, T, r_t)$, $T \geq t$, are arbitrage free discount factors at time t.

THREE In this problem use discount factors $\delta(t) = \exp(-.055t - .035t^2 + .010t^3)$ for times $0 \leq t \leq 2$.

- (a, 1 pts) Write the instantaneous spot rate $s(t)$.
(b, 2 pts) Find the instantaneous forward rate $f(t)$.

SOLUTION:

$$s(t) = -\frac{1}{t} \left(-.055t - .035t^2 + .010t^3 \right) = .055 + .035t - .010t^2, 0 < t \leq 2.$$

$$f(t) = -\frac{d}{dt} \left(-.055t - .035t^2 + .010t^3 \right) = .055t + .070t - .030t^2, 0 \leq t \leq 2$$

Math 325, Quiz 8B
October 31, 2008

PRINT NAME _____

Show enough work to support your answers, clearly indicate your answers. All rates are continuously compounded.

ONE In this problem use discount factors $\delta(t) = \exp(-.055t - .03t^2 + .005t^3)$ for times $0 \leq t \leq 2$.

(a, 2pts) Write the instantaneous spot rate curve $s(t)$.

(b, 2 pts) Find the instantaneous forward rate curve $f(t)$.

SOLUTION:

$$s(t) = -\frac{1}{t} \left(-.055t - .03t^2 + .005t^3 \right) = .055 + .030t - .005t^2, 0 < t \leq 2.$$

$$f(t) = -\frac{d}{dt} \left(-.055t - .030t^2 + .005t^3 \right) = .055t + .060t - .015t^2, 0 \leq t \leq 2$$

TWO (1 pt @) Is the statement TRUE or FALSE about every short rate model? (Circle your choice)

- (a) TRUE **FALSE** Each price $\$P(0, T, r_0)$, $T \geq 0$, is the current market prices of a par \$1 zero maturing in T years.
- (b) **TRUE** FALSE The prices $\$P(t, T, r_t)$, $T \geq t$, are arbitrage free prices for all par \$1 zeros maturing after time t.
- (c) TRUE **FALSE** $P(t, T, r) = P(0, T - t, r)$ for all values of r and $0 \leq t \leq T$.
- (d) TRUE **FALSE** The value of $P(2, 5, 0.07)$ must be $\exp(-0.21)$.

THREE (2 pts) If the forward rate is at least 4% at all times $0 \leq t \leq 3$, prove that the three year spot rate is at least 4%.

SOLUTION: If $f(t) \geq .04$ for all $0 \leq t \leq 3$ then $s(3) = \frac{1}{3} \int_0^3 f(u) du \geq \frac{1}{3} \int_0^3 0.04 du = .04$