Math 411, Quiz 7A PRINT NAME ____
October 24, 2008 Show enough work to support your answers and clearly indicate your answers.

ONE (2 pts @) Let X and Y have joint density $f(x, y) = 10x^2y$ for (x,y) in the triangle $0 < y < x < 1$, and f(x,y) = 0 otherwise. (a) Find the density of X. (b) Find the distribution of X. (c) Are X and Y independent? Explain briefly.

SOLUTIONS (a): $f_X(x) = \int_0^x 10x^2y\,dy = 5x^4$ for $0 \le x \le 1$, and $f_X(x) = 0$ otherwise.

(b): $F_X(x) = \int_0^x 5u^4\,du = x^5$ for $0 \le x \le 1$, $F_X(x) = 0$ for $x < 0$, and $F_X(x) = 1$ for $x > 1$.

(c): $f_Y(y) = \int_y^1 10x^2y\,dx = (10/3)(y - y^4)$ for $0 \le y \le 1$. X and Y are not independent because the joint density isn't the product of the marginal densities, i.e., $10x^2y \ne 5\ y^4(10/3)(y - y^4)$.

TWO (2 pts @) Let X and Y be independent, standard normal random variables.
(a) Write the joint density of X and Y. (b) Give $P(X > 0 \text{ and } Y < 0)$.

SOLUTIONS (a): Each density is $f(u) = (2\pi)^{-1/2}\exp\left(-u^2/2\right)$. By independence the joint density is

$$f(x, y) = f_X(x)f_Y(y) = (2\pi)^{-1/2}\exp\left(-u^2/2\right)(2\pi)^{-1/2}\exp\left(-u^2/2\right) = (2\pi)^{-1}\exp\left(-(x^2 + y^2)/2\right)$$

(b): By independence $P(X > 0 \text{ and } Y < 0) = P(X > 0)P(Y < 0) = (1/2)(1/2) = 1/4$

Math 411, Quiz 7B PRINT NAME ____
October 24, 2009 Show enough work to support your answers and clearly indicate your answers.

ONE (2 pts @) Let X and Y have joint density $f(x, y) = 2xy + x$ for (x,y) in the square $0 < y,x < 1$, and $f(x,y) = 0$ otherwise. (a) Find the density of X. (b) Find the distribution of X. (c) Are X and Y independent? Explain briefly.

SOLUTIONS (a): $f_X(x) = \int_0^1 (2xy + x)\,dy = 2x$.

(b) $F_X(x) = \int_0^x 2u\,du = x^2$ for $0 \le x \le 1$, $F_X(x) = 0$ for $x < 0$, and $F_X(x) = 1$ for $x > 1$.

(c): $f_Y(y) = \int_0^1 (2xy + x)\,dx = y + 1/2$. Since $f(x, y) = f_X(x) f_Y(y)$ the random variables are independent.

TWO (2 pts @) Let X and Y be independent, and both exponentially distributed with parameter $\lambda = 2$.
(a) Write the joint density of X and Y. (b) Give $P(X > 0 \text{ and } Y < 0)$.

SOLUTIONS (a) Each density is $f(u) = 2e^{-2u}$ for $u \ge 0$, and $f(u) = 0$ for $u < 0$. By independence the joint density is

$$f(x, y) = f_X(x) f_Y(y) = 2e^{-2y}\,2e^{-2y} = 4e^{-2(x+y)} .$$

(b): By independence $P(X > 0 \text{ and } Y < 0) = P(X > 0)P(Y < 0) = (1)(0) = 0$