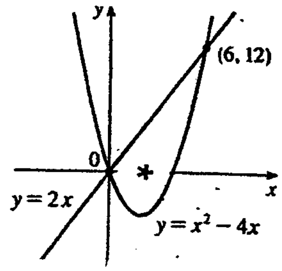
Math 172-504, Final Exam PRINT NAME ______________________________

May 7, 2008 **A** Show enough work to support your answers and clearly indicate them.

The region R in Problems 1 and 2 is marked with a star in the drawing.



1 (6 pts). Find the area of R.

$$\text{Area} = \int_0^6 \left[2x-(x^2-4x)\right]dx = 3x^2-\frac{1}{3}x^3\Big|_{x=0}^{x=6} = 36$$

2 (6 pts). Use the method of cylindrical shells to write an integral expression for the volume generated by rotating R about the y-axis.

$$\text{Volume} = \int_0^6 2\pi x\left[2x-(x^2-4x)\right]dx$$

3 (6 pts). Find $\int x\cos(x)dx$ Use parts, $u=x$, $dv=\cos(x)dx$, $du=dx$, $v=\sin(x)$

$$\int x\cos(x)dx = x\sin(x)-\int \sin(x)dx = x\sin(x)+\cos(x)+C$$

4 (6 pts). A force of 10 lbs is required to stretch a spring 4 inches beyond its natural length. How much work (in ft-lbs) is done in stretching it from its natural length to 6 inches beyond its natural length?

$$F=kx \Rightarrow 10=k(1/3) \Rightarrow k=30 \text{ lb/ft}$$

$$\text{Total Work} = \int_0^{1/2} 30x\,dx = 15/4 \text{ ft-lb}$$

5 (6 pts). Give the formula for $\int \tan(x)dx$; you need not derive the formula.

$$\int \tan x\,dx = \ln|\sec(x)|+C = -\ln|\cos(x)|+C$$

6 (6 pts). Write out the form of the partial fractions decomposition of $\frac{1}{x^2+x^4}$; do not determine the numerical values of the coefficients.

$$\frac{1}{x^2+x^4} = \frac{1}{x^2(1+x^2)} = \frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+1}$$

7 (6 pts). Solve the differential equation $y' + 2y = 4$.

$$\mu(x) = e^{\int 2dx} = e^{2x} \Rightarrow [e^{2x}y]' = e^{2x}y' + 2e^{2x}y = 4e^{2x} \Rightarrow$$

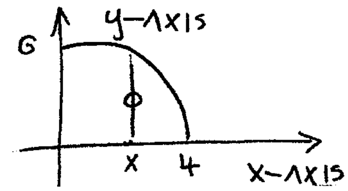
$$e^{2x}y = 2e^{2x} + c \Rightarrow y = 2 + ce^{-2x}$$

8 (6 pts). Write an integral expression for the area of the surface obtained by rotating $y = x^2$, $0 \le x \le 1$, about the x-axis.

$$\text{Surface Area} = \int_0^1 2\pi (x^2)\sqrt{1+(2x)^2}\, dx$$

9 (6 pts). Let M be the region in the first quadrant under the graph of $y = 3\sqrt{4-x}$. M has area 16. Write an integral expression for the x-coordinate of the centroid of M.

$$\bar{x} = \frac{1}{16}\int_0^4 x\, 3\sqrt{4-x}\, dx$$



10 (6 pts). Give the sum of the geometric series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^{2n}}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^{2n}} = \sum_{n=1}^{\infty} \frac{1}{9}\left(\frac{-1}{9}\right)^{n-1} = \frac{1}{9}\,\frac{1}{1-(-1/9)} = \frac{1}{10}$$

11 (6 pts). Does $\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$ converge or diverge? Name the test you use to decide.

Diverges by comparison with $\sum \frac{1}{n}$; $\frac{2n}{1+n^2} \Big/ \frac{1}{n} = \frac{2n^2}{n^2+1} \to 2$

Also diverges by integral test; $\int_1^n \frac{2x}{x^2+1} = \ln(x^2+1)\Big|_{x=1}^{x=n} = \ln\left(\frac{n^2+1}{2}\right) \to \infty$

12 (6 pts). Write the Maclaurin series for e^x. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

15. Each part refers to the plane $2x-y+2z=4$ and/or line $x=t, y=2t, z=1-t$.

(a, 2 pts) Give a normal vector for the plane. $\bar{n} = \langle 2,-1,2 \rangle$

(b, 2 pts) Write the symmetric equations for the line. $x = y/2 = 1-z$

(c, 2 pts) Find the point where the line and plane intersect.

$4 = 2(t)-(2t)+2(1-t) = 2-2t \Rightarrow t=-1 \Rightarrow$ point $= (-1,-2,2)$

(d, 2 pts) Give a direction vector for the line. $\bar{u} = \langle 1,2,-1 \rangle$

10. Find the area of the parallelogram with adjacent edges $a=i+j$ and $b=i-j+k$.

$\bar{a} \cdot \bar{b} = 0$ so the parallelogram is a rectangle with side lengths $|\bar{a}| = \sqrt{2}$ and $|\bar{b}| = \sqrt{3}$. Area $= \sqrt{2}\sqrt{3} = \sqrt{6}$

13. In each part find the indicated expression for the vectors $u = \langle 1,2,0 \rangle$ and $v = \langle 0,3,-4 \rangle$.

(a, 2 pts) $3u-v = \langle 3,6,0 \rangle - \langle 0,3,-4 \rangle = \langle 3,3,4 \rangle$

(b, 2 pts) $u \cdot v = 1(0)+2(3)+0(-4) = 6$

(c, 2 pts) $|u| = \sqrt{1^2+2^2+0^2} = \sqrt{5}$

(d, 2 pts) $u \times v = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & 0 \\ 0 & 3 & -4 \end{vmatrix} = \langle -8,4,3 \rangle$

14 (6 pts) Find the cosine of the angle between $a = \langle 1,2,2 \rangle$ and $b = \langle 3,4,0 \rangle$

$\bar{a} \cdot \bar{b} = |\bar{a}|\,|\bar{b}| \cos\theta \Rightarrow 11 = (3)(5)\cos\theta \Rightarrow \cos\theta = 11/15$