MATH 304 ASSIGNMENT 9

All problems are from Leon’s Linear Algebra, eighth edition, unless otherwise specified. Turning in extra problems will not result in any extra credit.

0. NOT TO BE TURNED IN

The following problems are meant to test your understanding but are not to be turned in.

- Section 5.2: Problems 1, 2, 3.

1. TO BE TURNED IN

Please complete these problems on a separate sheet of paper and hand them in.

1. (Section 5.2, problem 1d) Suppose $A$ is the following matrix:

$$
A = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 \\
1 & 1 & 2 & 2
\end{pmatrix}
$$

Determine a basis for each of the subspaces $C(A^T)$, $N(A)$, $C(A)$, and $N(A^T)$.

2. (Section 5.2, problem 4) Let $S$ be the subspace of $\mathbb{R}^4$ spanned by

$$
x_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \end{pmatrix} \quad \text{and} \quad x_2 = \begin{pmatrix} 0 \\ 1 \\ 3 \\ -2 \end{pmatrix}
$$

Find a basis for $S^\perp$.

3. (Section 5.2, problem 6) Is it possible for a matrix to have the vector $(3, 1, 2)$ in its row space and the vector $(2, 1, 1)^\top$ in its null space? Explain.

4. (Section 5.2, problem 8) Let $S$ be the subspace of $\mathbb{R}^n$ spanned by the vectors $x_1, x_2, \ldots, x_k$. Show that $y \in S^\perp$ if and only if $y \perp x_i$ for $i = 1, \ldots, k$.

5. (Section 5.2, problem 9) If $A$ is an $m \times n$ matrix of rank $r$, what are the dimensions of $N(A)$ and $N(A^\top)$? Explain.

6. (Section 5.2, problem 15) Let $U$ and $V$ be subspaces of a vector space $W$. Show that if $W = U \oplus V$, then $U \cap V = \{0\}$.

7. (Section 5.2, problem 17) Let $x$ and $y$ be linearly independent vectors in $\mathbb{R}^n$ and let $S = \text{Span}(x,y)$. We can use $x$ and $y$ to define an $n \times n$ matrix $A$ by setting

$$
A = xy^\top + yx^\top
$$

(a) Show that $A$ is symmetric.

(b) Show that $N(A) = S^\perp$.

(c) Show that the rank of $A$ must be 2.

Date: 1 April 2015.
2. Extra practice

These problems are suggested in case you would like extra practice with the concepts from class. They are not to be turned in.

- Section 5.2: Problems 5, 7, 11, 13, 14, 16