MATH 304 ASSIGNMENT 13

All problems are from Leon’s Linear Algebra, eighth edition, unless otherwise specified. Turning in extra problems will not result in any extra credit.

0. NOT TO BE TURNED IN

The following problems are meant to test your understanding but are not to be turned in.

- Section 6.1: Problems 1, 2, 3, 4, 13.
- Section 6.3: Problems 1, 3, 4, 8.

1. TO BE TURNED IN

Please complete these problems on a separate sheet of paper and hand them in.

1. (Section 6.1, problem 1i) Find the eigenvalues and the corresponding eigenvectors for the following matrix:

\[
\begin{pmatrix}
4 & -5 & 1 \\
1 & 0 & -1 \\
0 & 1 & -1
\end{pmatrix}
\]

2. (Section 6.1, problem 7) Let \( A \) be an \( n \times n \) matrix and let \( B = I - 2A + A^2 \).
   (a) Show that if \( x \) is an eigenvector of \( A \) belonging to an eigenvalue \( \lambda \) of \( A \), then \( x \) is also an eigenvector of \( B \) belonging to an eigenvalue \( \mu \) of \( B \). How are \( \mu \) and \( \lambda \) related?
   (b) Show that if \( \lambda = 1 \) is an eigenvalue of \( A \), then the matrix \( B \) will be singular.

3. (Section 6.1, problem 14) Let \( A \) be a \( 2 \times 2 \) matrix. If the trace of \( A \) (the sum of the diagonal entries of \( A \)) is 8 and the determinant of \( A \) is 12, what are the eigenvalues of \( A \)?

4. (Section 6.3, problems 1e and 3e) Let \( A \) be the following matrix:

\[
A = \begin{pmatrix}
1 & 0 & 0 \\
-2 & 1 & 3 \\
1 & 1 & -1
\end{pmatrix}
\]
   (a) Factor the matrix \( A \) into a product \( XDX^{-1} \), where \( D \) is diagonal.
   (b) Use the factorization above to compute \( A^{-1} \), if it exists.

5. (Section 6.3, problem 6) Let \( A \) be a diagonalizable matrix whose eigenvalues are all either \( 1 \) or \( -1 \). Show that \( A^{-1} = A \).

6. (Section 6.3, problem 7) Show that any \( 3 \times 3 \) matrix of the form

\[
\begin{pmatrix}
a & 1 & 0 \\
0 & a & 1 \\
0 & 0 & b
\end{pmatrix}
\]

is defective.

Date: 29 April 2015.
7. (Section 6.3, problem 8d) Find all possible values of the scalar $\alpha$ so that the following matrix $A$ is defective or show that no such values exist.

$$A = \begin{pmatrix} 4 & 6 & -2 \\ -1 & -1 & 1 \\ 0 & 0 & \alpha \end{pmatrix}$$

2. **Extra practice**

These problems are suggested in case you would like extra practice with the concepts from class. They are not to be turned in.

- Section 6.1: Problems 8, 10, 12, 27, 28.
- Section 6.3: Problems 5, 9, 12.