MATH 304 ASSIGNMENT 5

All problems are from Leon’s *Linear Algebra*, eighth edition, unless otherwise specified. Turning in extra problems will *not* result in any extra credit.

0. **NOT TO BE TURNED IN**

The following problems are meant to test your understanding but are not to be turned in.
- Section 3.2: 1, 2, 3, 4, 14.

1. **TO BE TURNED IN**

Please complete these problems on a separate sheet of paper and hand them in.

1. (Section 3.2, problem 1(d)): Determine whether the following set is a subspace of $\mathbb{R}^2$:
   \[
   \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} : |x_1| = |x_2| \right\}
   \]
   If it is a subspace, show it is. If not, explain why not.

2. (Section 3.2, problem 2(c)): Determine whether the following set is a subspace of $\mathbb{R}^3$:
   \[
   \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} : x_3 = x_1 + x_2 \right\}
   \]
   If it is a subspace, show it is. If not, explain why not.

3. (Section 3.2, problem 4(d)): Determine the null space of the following matrix:
   \[
   \begin{pmatrix}
   1 & 1 & -1 & 2 \\
   2 & 2 & -3 & 1 \\
   -1 & -1 & 0 & -5
   \end{pmatrix}
   \]

4. (Section 3.2, problem 8): Let $A$ be a fixed matrix in $\mathbb{R}^{n \times n}$ and let $S$ be the set of all matrices that commute with $A$, i.e.,
   \[
   S = \left\{ B \in \mathbb{R}^{n \times n} : AB = BA \right\}.
   \]
   Show that $S$ is a subspace of $\mathbb{R}^{n \times n}$.

5. (Section 3.2, problem 11): Determine whether the following are spanning sets for $\mathbb{R}^2$:
   \[
   \begin{align*}
   &\text{(a)} \quad \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\} \\
   &\text{(b)} \quad \left\{ \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \end{pmatrix} \right\} \\
   &\text{(c)} \quad \left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \end{pmatrix} \right\} \\
   &\text{(d)} \quad \left\{ \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ -4 \end{pmatrix} \right\}
   \end{align*}
   \]

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(e) \( \left\{ \left( \frac{1}{2}, \frac{-1}{1} \right) \right\} \)

6. (Section 3.2, problem 20): Let \( U \) and \( V \) be subspaces of a vector space \( W \). Prove that their intersection \( U \cap V \) is also a subspace of \( W \).

7. (Section 3.2, problem 21): Let \( S \) be the subspace of \( \mathbb{R}^2 \) spanned by \( e_1 \) and let \( T \) be the subspace of \( \mathbb{R}^2 \) spanned by \( e_2 \). Is \( S \cup T \) a subspace of \( \mathbb{R}^2 \)? Explain.

2. Extra practice

These problems are suggested in case you would like extra practice with the concepts from class. They are not to be turned in.

- Section 3.2: Problems 5, 6, 7, 10, 12, 15, 22.