MATH 304 ASSIGNMENT 7

All problems are from Leon’s *Linear Algebra*, eighth edition, unless otherwise specified. Turning in extra problems will *not* result in any extra credit.

0. NOT TO BE TURNED IN

The following problems are meant to test your understanding but are not to be turned in.

- Section 3.5: Problems 1, 2, 3, 4.
- Section 3.6: Problems 1, 2, 3, 7, 10.

1. TO BE TURNED IN

Please complete these problems on a separate sheet of paper and hand them in.

1. (Section 3.5, problem 5) Let \( u_1, u_2, \) and \( u_3 \) be given as follows:

\[
\begin{align*}
  u_1 &= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, &
  u_2 &= \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, &
  u_3 &= \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}
\end{align*}
\]

(a) Find the transition matrix corresponding to the change of basis from \( \{e_1, e_2, e_3\} \) to \( \{u_1, u_2, u_3\} \).

(b) Find the coordinates of each of the following vectors with respect to \( \{u_1, u_2, u_3\} \).

(i) \( \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \)

(ii) \( \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \)

(iii) \( \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \)

2. (Section 3.5, problem 8) Given

\[
\begin{align*}
  v_1 &= \begin{pmatrix} 2 \\ 6 \end{pmatrix}, &
  v_2 &= \begin{pmatrix} 1 \\ 4 \end{pmatrix}, &
  S &= \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix}
\end{align*}
\]

Find vectors \( u_1 \) and \( u_2 \) so that \( S \) will be the transition matrix from \( \{v_1, v_2\} \) to \( \{u_1, u_2\} \).

3. (Section 3.5, problem 10) Find the transition matrix representing the change of coordinates on \( P_3 \) (the space of polynomials of degree less than 3) from the ordered basis \([1, x, x^2]\) to the ordered basis

\( [1, 1 + x, 1 + x + x^2] \)

Date: 11 March 2015.
4. (Section 3.6, problem 2c) Determine the dimension of the subspace of $\mathbb{R}^3$ spanned by the following vectors:

\[
\begin{pmatrix}
1 \\
-1 \\
2
\end{pmatrix}, \quad
\begin{pmatrix}
-2 \\
2 \\
4
\end{pmatrix}, \quad
\begin{pmatrix}
3 \\
-2 \\
5
\end{pmatrix}, \quad
\begin{pmatrix}
2 \\
-1 \\
3
\end{pmatrix}
\]

5. (Section 3.6, problem 13) Let $A$ be a $4 \times 3$ matrix and suppose that the vectors

\[
z_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad z_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}
\]

form a basis for $N(A)$. If $b = a_1 + 2a_2 + a_3$ (where $a_j$ are the columns of $A$), find all solutions of the system $Ax = b$.

6. (Section 3.6, problem 15) Let $A$ be a $4 \times 5$ matrix (with columns $a_j$) and let $U$ be the reduced row echelon form of $A$. If

\[
a_1 = \begin{pmatrix}
2 \\
1 \\
-3 \\
-2
\end{pmatrix}, \quad a_2 = \begin{pmatrix}
-1 \\
2 \\
3 \\
1
\end{pmatrix}, \quad U = \begin{pmatrix}
1 & 0 & 2 & 0 & -1 \\
0 & 1 & 3 & 0 & -2 \\
0 & 0 & 0 & 1 & 5 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

(a) find a basis for $N(A)$
(b) given that $x_0$ is a solution of $Ax = b$, where

\[
b = \begin{pmatrix}
0 \\
5 \\
3 \\
4
\end{pmatrix} \quad \text{and} \quad x_0 = \begin{pmatrix}
3 \\
2 \\
0 \\
2 \\
0
\end{pmatrix}
\]

(i) find all solutions to the system.
(ii) determine the remaining column vectors of $A$.

7. (Section 3.6, problem 16) Let $A$ be any $5 \times 8$ matrix with rank equal to 5 and let $b$ be any vector in $\mathbb{R}^5$. Explain why the system $Ax = b$ must have infinitely many solutions.

2. Extra practice

These problems are suggested in case you would like extra practice with the concepts from class. They are not to be turned in.

- Section 3.5: Problems 6, 7, 8.
- Section 3.6: Problems 4, 12, 18, 21, 25, 27, 29, 31.