1. For each of the following matrices, determine whether the matrix is in reduced row echelon form (Y) or not (N).

   (a) (1 point) \[
   \begin{pmatrix}
   1 & 0 & 2 & 0 & 1 \\
   0 & 1 & 0 & 0 & 0
   \end{pmatrix}
   \]

   (a) \(\text{Y}\)

   (b) (1 point) \[
   \begin{pmatrix}
   1 & 0 & 2 \\
   0 & 1 & 0
   \end{pmatrix}
   \]

   (b) \(\text{Y}\)

   (c) (1 point) \[
   \begin{pmatrix}
   1 & 1 & 1 \\
   0 & 1 & 0 \\
   0 & 0 & 0
   \end{pmatrix}
   \]

   (c) \(\text{N}\)

2. (2 points) Suppose \(A\) is an \(m \times n\) matrix and \(B\) is an \(n \times r\) matrix. How many rows will the matrix \((B^tA^t)^t\) have?

   2. \(m\)

3. (5 points) Find all solutions of the following system of equations by reducing the corresponding augmented matrix to reduced row echelon form.

   \[
   \begin{align*}
   x_1 - x_2 - 2x_3 &= -2 \\
   3x_1 + x_2 + 2x_3 &= 6 \\
   4x_1 + 2x_2 + 4x_3 &= 10
   \end{align*}
   \]

   **Solution:** The corresponding augmented matrix is given by the following:

   \[
   \begin{pmatrix}
   1 & -1 & -2 & -2 \\
   3 & 1 & 2 & 6 \\
   4 & 2 & 4 & 10
   \end{pmatrix}
   \]

   Replacing \(R_2\) by \(R_2 - 3R_1\) and \(R_3\) by \(R_3 - 4R_1\), we obtain the following:

   \[
   \begin{pmatrix}
   1 & -1 & -2 & -2 \\
   0 & 4 & 8 & 12 \\
   0 & 6 & 12 & 18
   \end{pmatrix}
   \]

   Dividing the second row by 4 we obtain:

   \[
   \begin{pmatrix}
   1 & -1 & -2 & -2 \\
   0 & 1 & 2 & 3 \\
   0 & 6 & 12 & 18
   \end{pmatrix}
   \]
We now replace $R_3$ by $R_3 - 6R_2$ and replace $R_1$ by $R_1 + R_2$, yielding:

$$
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0
\end{pmatrix}
$$

This matrix is in reduced row echelon form. The first equation corresponds to $x_1 = 1$, while the second equation corresponds to $x_2 + 2x_3 = 3$. This means that $x_3$ is a free variable and all solutions are given by

$$
\{(1, 3 - 2x_3, x_3) : x_3 \in \mathbb{R}\}.
$$