1. Determine whether each of the following statements is true (T) or false (F).

(a) (2 points) Any three vectors in $\mathbb{R}^4$ must be linearly independent.  
(a) $\text{F}$

(b) (2 points) Any four vectors in $\mathbb{R}^3$ must be linearly dependent.  
(b) $\text{T}$

(c) (2 points) A basis for $\mathbb{R}^n$ must have exactly $n$ vectors.  
(c) $\text{T}$

2. (4 points) Show that

\[
\begin{pmatrix}
1 & 1 & -2 \\
0 & 2 & 2 \\
1 & 3 & 1
\end{pmatrix}
\]

is a basis for $\mathbb{R}^3$.

**Solution:** Because $\dim \mathbb{R}^3 = 3$, we must only show that the three vectors are linearly independent. Because

\[
\det \begin{pmatrix}
1 & 1 & -2 \\
0 & 2 & 2 \\
1 & 3 & 1
\end{pmatrix} = 1(2 - 6) - 0(1 + 6) + 1(2 + 4) = -4 + 6 = 2 \neq 0,
\]

this matrix is nonsingular and so its columns (the vectors in question) are linearly independent.