1. Consider the following differential equation:

\[ y'' + 9y' + 8y = g(t). \]

Find the general solution of the inhomogeneous equation when:

(a) \( g(t) = 60 \sin(2t) \)

**Solution:** The characteristic equation for the homogeneous equation \( y'' + 9y' + 8y = 0 \) is \( r^2 + 9r + 8 = 0 \), which has roots \( r = -8 \) and \( r = -1 \). The general solution of the homogeneous problem is thus

\[ y_h = c_1 e^{-t} + c_2 e^{-8t}. \]

Use method of undetermined coefficients, make guess

\[ y_p = A \sin(2t) + B \cos(2t), \]

so that

\[ y'_p = 2A \cos(2t) - 2B \sin(2t) \]
\[ y''_p = -4A \sin(2t) - 4B \cos(2t) \]

Plugging our guess into the equation yields:

\[ (-4A - 18B + 8A) \sin(2t) + (-4B + 18A + 8B) \cos(2t) = 60 \sin(2t), \]

i.e., we want \( A \) and \( B \) to satisfy

\[ 4A - 18B = 60 \]
\[ 18A + 4B = 0, \]

so we have \( A = 12/17 \) and \( B = -54/17 \). The general solution is thus

\[ y = \frac{12}{17} \sin(2t) - \frac{54}{17} \cos(2t) + c_1 e^{-8t} + c_2 e^{-t}. \]

(b) \( g(t) = e^{-t} \)

**Solution:** \( e^{-t} \) is a solution of the homogeneous equation, so we modify our guess to \( y_p = Ate^{-t} \), so that

\[ y'_p = Ae^{-t} - Ate^{-t} \]
\[ y''_p = Ate^{-t} - 2Ae^{-t} \]
and so, plugging back into the equation, we have

\[(A - 9A + 8A)te^{-t} + (-2A + 9A)e^{-t} = e^{-t},\]

i.e., we want \(7A = 1\), so \(y = \frac{1}{7}te^{-t}\) is a solution, and the general solution is

\[y = \frac{1}{7}te^{-t} + c_1e^{-t} + c_2e^{-8t}\]

---

2. A mass of 100g stretches a spring 5cm. If the mass is set in motion from its equilibrium position with a downward velocity of 10 cm/s and if there is no damping, write a differential equation (with initial values) describing the position \(u\) of the mass at any time \(t\).

**Solution:** The mass stretches the string 5cm, so we must have \(5k = 100(g)\), where \(g = 980\text{cm/s}^2\). We thus have \(k = 20 \times 980\). No damping means that \(\gamma = 0\) and so the differential equation describing the position \(u\) is

\[mu'' + ku = 0,\]

where \(k\) is as above.

For the initial conditions, it is set in motion from its equilibrium position, so \(u(0) = 0\), and it has initial velocity \(u'(0) = 10\text{cm/s}\).