1. Consider the following initial value problem with impulsive forcing:

\[
y'' + 4y = \delta(t - 4) \\
y(0) = 0 \\
y'(0) = 0
\]

Find an expression for the Laplace transform \(Y(s)\) of \(y(t)\). (You do not have to find the inverse Laplace transform of your answer.)

**Solution:** Take the Laplace transform of both sides to find

\[
s^2Y(s) + 4Y(s) = e^{-4s},
\]

so that

\[
Y(s) = \frac{e^{-4s}}{s^2 + 4}.
\]

2. Suppose \(f(t) = 1\) and \(g(t) = t\). Compute the convolution product \((f \ast g)(t)\).

**Solution:** We compute:

\[
(f \ast g)(t) = \int_0^t f(\tau)g(t - \tau)\,d\tau \\
= \int_0^t (t - \tau)\,d\tau \\
= (t\tau - \frac{1}{2}\tau^2)\big|_{\tau=0}^{\tau=t} = t^2 - \frac{1}{2}t^2 = \frac{1}{2}t^2.
\]