# 1 HOMEWORK #1. Solutions of the problems that will be graded

The five problems that will be graded (out of the nine problems that you had to turn in) are :
Section 1.3, Problem #26; Sec. 1.5, Prob. #24; Sec. 1.6, Prob. #33; Sec. 1.6, Prob. #34; Sec 1.9, Prob. #23;

Below you can find the solutions of those 5 problems.

## 1.1 Section 1.3, Problem 26

The table below gives the sales, S, of Intel Corporation (in millions of dollars)

| Year | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 |
|---|---|---|---|---|---|---|
| S | 26273 | 29389 | 33726 | 26539 | 26764 | 30141 |

**(a)** Find the change in sales between 1998 and 2003.
**(b)** Find the average rate of change in sales between 1998 and 2003. Give units and interpret your answer.
**(c)** If the average rate of change in sales continues at the same rate as between 2001 and 2003, in which year will the sales first reach 40000 million dollars?
**Solution:**
**(a)** change in sales = sales in 2003 - sales in 1998 i.e. 30141 - 26273 = 3868 million dollars.
**(b)**

$$\text{average rate of change} = \frac{\text{change in sales}}{\text{change in time}} = \frac{3868}{2003-1998} = \frac{3868}{5} = 773.6 \text{ million dollars per year.}$$

This means that between 1998 and 2003 the Intel sales increased on average by $773.6 million per year.
**(c)** As above we find that the average rate of change in sales between 2001 and 2003 is:

$$\frac{30141-26539}{2003-2001} = \frac{3602}{2} = 1801$$

million dollars per year. Assuming this rate continues, the sales $t$ years after 2003 will be $S(t) = 30141 + 1801t$. We want to know when will the sales reach 40000

$$40000 = 30141 + 1801t \Rightarrow t \approx 5.47$$

and therefore $t$he sales are expected to reach 40000 million dollars in 2009 (2003+6). Indeed the formula above tells us that the sales in 2008 will be $30141 + 1801 \cdot 5 = 39146$ million dollars, and the sales in 2009 will be $30141 + 1801 \cdot 6 = 40947$ million dollars.

## 1.2 Section 1.5, Problem #24

The problem: **(a)** Niki invested $10000 in the stock market. The investment was a loser, declining in value 10% per year each year for 10 years. How much was the investment worth after 10 years?
**(b)** After 10 years, the stock began to gain value at 10% per year. After how long will the investment regain its original value ($10000)?
**Solution:**
**(a)** The formula that will give the value of the investment for the first 10 years is $Q(t) = Q_0(1-r)^t = 10000(0.9)^t$. Thus after 10 years the value of the investment will be $10000(0.9)^{10} \approx 3486.78$ dollars.
**(b)** The formula that will give the value of the investment after the first 10 years is $P(t) = P_0(1+r)^t = 3486.78(1.1)^t$. Solving the equation

$$10000 = 3486.78(1.1)^t$$

for the unknown $t$ we get

$$t = \frac{\ln \frac{10000}{3486.78}}{\ln 1.1} \approx 11.05,$$

so the investment will regain its original value in approximately 11 years.

### 1.3 Section 1.6, Problem #33

The problem: **(a)** What is the continuous percent growth rate for $P = 100e^{0.06t}$, with time, $t$, in years ?
**(b)** Write this function in the form $P = P_0 a^t$. What is the annual percent growth rate?

**Solution:**
**(a)** The continuous percent growth rate is 6%.
**(b)**

$$100e^{0.06t} = 100(e^{0.06})^t \approx 100(1.0618)^t$$

The (discrete) annual percent growth rate is $\approx 6.18\%$.

### 1.4 Section 1.6, Problem #34

The problem: **(a)** What is the annual percent decay rate for $P = 25(0.88)^t$, with time, $t$, in years ?
**(b)** Write this function in the form $P = P_0 e^{kt}$. What is the continuous percent growth rate?

**Solution:**
**(a)** The annual percent decay rate is $(1 - 0.88) = 12\%$.
**(b)**

$$25(0.88)^t = 25(e^{\ln 0.88})^t \approx 25e^{(-0.1278)t}$$

The continuous percent decay rate is $\approx 12.78\%$.

### 1.5 Section 1.9, Problem #23

The circulation time of a mammal (that is, the average time it takes for all the blood in the body to circulate once and return to the heart) is proportional to the fourth root of the body mass of the mammal.

**(a)** Write a formula for the circulation time, $T$, in terms of the body mass, $B$.
**Answer:** $T = k\sqrt[4]{B}$

**(b)** If an elephant of body mass $5230kg$ has a circulation time of $148s$, find the constant of proportionality.
**Answer:**

$$k = \frac{T}{\sqrt[4]{B}} \approx \frac{148}{8.5040} \approx 17.40$$

**(c)** What is the circulation time of of a human with body mass $70kg$.
**Answer:**

$$T = 17.4\sqrt[4]{70} \approx 17.4(2.8925) \approx 50.3$$