

# 1 HOMEWORK #2. Solutions of the problems that will be graded

The five problems that will be graded (out of the eight problems that you had to turn in) are :  
Section 2.3, Problem #9, Prob. #22; Sec. 2.4, Prob. #10; Sec. 3.1, Prob. #19; Prob. #43;

Below you can find the solutions of those 5 problems.

## 1.1 Section 2.3, Problem 9

The weight,  $W$ , in lbs. of a child is a function of its age,  $a$ , in years, so  $W = f(a)$ .

- (a) Do you expect  $f'(a)$  to be positive or negative? Why?
- (b) What does  $f(8) = 45$  tell you? Give units for the numbers 8 and 45.
- (c) What are the units of  $f'(a)$ ? Explain what  $f'(a)$  tells you in terms of age and weight.
- (d) What does  $f'(8) = 4$  tell you in terms of age and weight.
- (e) If  $a$  increases, do you expect  $f'(a)$  to increase or decrease? Explain.

### Answers:

- (a) Positive, since weight increases as the child gets older.
- (b)  $f(8) = 45$  tells us that when the child is 8 years old, the child weighs 45 pounds.
- (c) The units of  $f'(a)$  are lbs/year (pounds per year)?  $f'(a)$  tells the rate of growth (in lbs/year) of the child at age  $a$ .
- (d)  $f'(8) = 4$  tells us that the 8-year-old child is growing at about 4 lbs/year.
- (e) As  $a$  increases,  $f'(a)$  will decrease, since the rate of growth slows down as the child grows up.

## 1.2 Section 2.3, Problem 22

Suppose  $f(x)$  is a function with  $f(20) = 345$  and  $f'(20) = 6$ . Estimate  $f(22)$ .

### Answer:

Assuming that the rate of change of  $f(x)$  stays the same for the interval  $(20, 22)$  we have:

$$6 = f'(20) = \frac{f(22) - f(20)}{22 - 20} = \frac{f(22) - 345}{2} \text{ and so we estimate } f(22) = 345 + 2 \cdot 6 = 357$$

## 1.3 Section 2.4, Problem #10

This problem deals with the graph on page 123 of your books.

- (a) Estimate the intervals on which the derivative is positive and the intervals on which the derivative is negative.
- (b) Estimate the intervals on which the second derivative is positive and the intervals on which the second derivative is negative.

### Answers:

- (a) The derivative is positive on those intervals where the function is increasing. The derivative is negative on those intervals where the function is decreasing. Therefore the derivative will be positive on the intervals  $0 < t < 0.4$  and  $1.7 < t < 3.4$  and negative on the intervals  $0.4 < t < 1.7$  and  $3.4 < t < 4$ .
- (b) The second derivative is positive on those intervals where the function is concave up. The second derivative is negative on those intervals where the function is concave down. Therefore the second derivative will be positive on the interval  $1 < t < 2.6$  and negative on the intervals  $0 < t < 1$  and  $2.6 < t < 4$ .

#### 1.4 Section 3.1, Problem #9

Find the derivative of  $y = \frac{1}{r^{7/2}}$  ?

**Solution:**

$$y = \frac{1}{r^{7/2}} = r^{-7/2} \Rightarrow y' = -\frac{7}{2}r^{-7/2-1} = -\frac{7}{2}r^{-9/2} = -3.5\frac{1}{r^{4.5}}$$

#### 1.5 Section 3.1, Problem #43

If  $f(t) = t^4 - 3t^2 + 5t$ , find  $f'(t)$  and  $f''(t)$ .

**Answer:**

$$f(t) = t^4 - 3t^2 + 5t \Rightarrow f'(t) = 4t^3 - 6t + 5. \quad f''(t) = (f'(t))' = 12t^2 - 6.$$