MATH 131
Summer 2006, Instructor : Dimitar Trenev — Name : ....SOLUTIONS............
Quiz # 3 — Section : 200

Problem #1. *(2 points)* The derivative of $f(x) = \ln(1-x)$ is :
(a) $\frac{1}{1-x}$ (b) $\frac{1}{x}$ (c) $\frac{1}{\ln(x)}$ (d) $\frac{-1}{\ln(1-x)}$ (e) $\frac{-1}{1-x}$

Answer: The derivative of $\ln(x)$ is $\frac{1}{x}$ and the derivative of $(1-x)$ is -1, so according to the chain rule, we will have

$$f'(x) = \frac{1}{1-x}(-1) = \frac{-1}{1-x}$$

Problem #2. *(3 points)* The equation of the line tangent to $f(x) = x^3 + 4x - 2$ at the point $(1,3)$:
(a) $y = 7x - 4$ (b) $y = 5x - 2$ (c) $y = (3x^2 + 4)(x-1) + 3$ (d) $y = 7x + 3$ (e) $y = 5x + 2$

Answer: Since $f'(x) = 3x^2 + 4$ (and the slope of the tangent line at $x = 1$ is the value of the derivative there), we have $m = f'(1) = 3(1)^2 + 4 = 7$. Using the point-slope formula for the point $(1,3)$ and a slope of 7, we get

$$y - 3 = 7(x-1),$$

which is the same as

$$y = 7x - 4$$

Problem #3. Consider the function $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - 2x + 5$.

(a) *(3 points)* Find the critical and inflection points of $f(x)$.
Solution:
Since $f'(x) = x^2 + x - 2 = (x+2)(x-1)$ we find that the critical points of $f(x)$ (i.e. the points where $f'(x)$ is 0 or non-existant) are the points $x = -2$ and $x = 1$.

(b) *(2 points)* Find the global maximum of $f(x)$ in the interval [-3,2].
Solution:
To find the global extrema of $f(x)$ we need to evaluate the function at the endpoints of our interval and at the critical points inside the interval.

$$f(-3) = \frac{13}{2} = 6.5, \quad f(-2) = \frac{25}{3} \approx 8.33, \quad f(1) = \frac{23}{6} \approx 3.83 \quad \text{and} \quad f(2) = \frac{17}{3} \approx 5.66,$$

from where we see that the maximum (≈ 8.33) is attained at $x = -2$ (and the minimum (≈ 3.83) at $x = 1$).