MATH 131
Summer 2006, Instructor : Dimitar Trenev — Name : ....
Quiz # 4 — Section : 200

Problem #1. *(3 points)* If $\int_{-1}^{5} f'(t)dt = 10$ and $f(-1) = -30$, then $f(5)$ is:
(a) -40 (b) -30 (c) -20 (d) 20 (e) 40

Answer:
According to the fundamental theorem, $10 = \int_{-1}^{5} f'(t)dt = f(5) - f(-1)$ and so $f(5) = f(-1) + 10 = -30 + 10 = -20$.

Problem #2. *(2 points)* The area between the $x-$axis and $y = x^2 - 1$ is:
(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$ (e) $\frac{7}{6}$

Solution:
The function $x^2 - 1 = (x-1)(x+1)$ intersects the $x-$axis at -1 and 1 and the area between it and the $x-$axis lies under the $x-$axis, so it is given by $A = -\int_{-1}^{1}(x^2-1)dx$. You can evaluate that integral on your calculator, or by using the fundamental teorem.
Since $F(x) = \frac{x^3}{3} - x$ is an antiderivative for $x^2 - 1$, according to the fundamental theorem :

$$-\int_{-1}^{1}(x^2-1)dx = -(F(1) - F(-1)) = -((\frac{1}{3} - 1) - (-\frac{1}{3} + 1)) = \frac{4}{3}$$

Problem #3. Find the integral:

$$\int \frac{1+e^x}{\sqrt{x+e^x}}dx.$$

(Do not simply put an asnwer! Either verify your answer by differentiation, or clearly show how you arrived at it).

Solution:
If we set $u = x + e^x$, then $du = (1+e^x)dx$ and:

$$\int \frac{1+e^x}{\sqrt{x+e^x}}dx == \int \frac{1}{\sqrt{u}}du = \int u^{-\frac{1}{2}}du = \frac{1}{(-\frac{1}{2}+1)}u^{-\frac{1}{2}+1} = 2u^{\frac{1}{2}} = 2\sqrt{x+e^x}$$