# Math 151 Sample Test III Questions Mon, 21/Nov/2005

Short answers to the sample problems for Test 3
(Sorry, I didn't have time to write down long solutions)
((Also, I may have made a mistake somewhere in the rush!))

1. Let $f(x) = x^{(-3/x^4)}$. Find $f'(x)$ and $\lim_{x\to\infty} f(x)$.

   **Answer :** Using logarithmic differentiation we find $\frac{f'(x)}{f(x)} = (\log f(x))' = \frac{12\log x - 3}{x^5}$ from which we get $f'(x) == (\frac{12\log x-3}{x^5})x^{(\frac{-3}{x^4})}$. For the limit, using L'Hospital rule we find that $\log f(x) \to 0$ as $x$ goes to infinity and therefore $f(x) \to e^0 = 1$

2. Evaluate the following limits.

   (a) $\lim_{x\to\infty} \dfrac{e^{2x}}{e^x + e^{-x}}$. **Answer:** $\infty$ (divide top and bottom by $e^{2x}$)

   (b) $\lim_{x\to\infty} \dfrac{\ln(\ln x)}{\ln x}$. **Answer:** 0 (L'Hospital)

   (c) $\lim_{x\to 1^+} (\ln x)^{\cos x}$. **Answer:** 0 (just evaluate)

   (d) $\lim_{x\to 0} \dfrac{x - \sin^{-1} 2x}{x^2 + \tan^{-1} x}$. **Answer:** $-1$ (L'Hospital)

3. Find the antiderivatives of $f(x) = 3^x + x$ and $g(x) = \frac{x+1}{x}$
   **Answer:** $F(x) = (\log 3)3^x + \frac{1}{2}x^2 + C_f$ and $G(x) = x + \log x + C_g$.

4. Yelnick McWawa invests \$1000. At the end of 5 years, the value of the investment is \$2000. What was his yearly interest rate if the interest was compounded continuously? If the interest rate does not change, what will the value of the investment be 21 years from when he started investing?
   **Answer:** Continuously compounded interest satisfies $A(t) = A_0 e^{nt}$, where $A_0$ is the amount initially invested (in our case 1000) and $n$ is the yearly iterest rate. Using $A(5) = 1000$ we find $n = \frac{\log 2}{5}$ and then $A(21) = 1000(2^{\frac{21}{5}}) \approx 18379$.

5. Find the domain, asymptotes, intervals of increase or decrease, and intervals of concavity for $f(x) = \dfrac{\sqrt{1-x}}{x}$.
   **Answer:** Domain = $\{x \le 1 | x \ne 0\}$. Asymptotes: horizontal at $y = 0$, vertical at $x = 0$. First derivative $f'(x) = \frac{x-2}{2x^2\sqrt{1-x}}$ is always negative in the domain, so our function is decreasing (plot it!). Second derivative $f''(x) = \frac{3x^2-12x+8}{4x^3\sqrt{1-x}^3}$ is negative on $(-\infty, 0)$, positive on $(0, 2 - \frac{2\sqrt{3}}{3})$, negative on $(2 - \frac{2\sqrt{3}}{3}, 1)$ so our function is concave down, concave up and concave down again (on those 3 intervals respectively).

6. A wire 50” long is cut into two pieces of possibly different lengths, one piece bent into an equilateral triangle, and the other bent into a circle. How long must each piece be to minimize the total area enclosed by the circle and triangle?
**Answer:** Call $r$ the radius of the cricle and $a$ the side of the triangle. We want to minimize the area $A = \pi r^2 + \frac{\sqrt{3}a^2}{4}$ when we have $2\pi r + 3a = 50$. Writing it in terms of finding a minimum of a function on a given interval, we want to minimize $A(a) = \frac{(50-3a)^2}{4\pi} + \frac{\sqrt{3}a^2}{4}$ on $(0, \frac{50}{3})$. $A'(a) = \frac{1}{4\pi}(2(9+\sqrt{3}\pi)a - 300)$ changes sign $(- \to +)$ at $a = \frac{150}{9+\sqrt{3}\pi}$, which is in our interval ($\frac{50}{3} = \frac{150}{9} > \frac{150}{9+\sqrt{3}\pi}$) and that's where the minimum is attained.

7. A car braked with a constant deceleration of $36\frac{ft}{s^2}$, producing skid marks measuring 112.5 ft before coming to a stop. How fast (in $\frac{ft}{s}$) was the car travelling when the breaks were first applied?
**Answer:** Problem taken directly from Fall’01 Exam 3 (look up the solution on the web)

8. A box with an open top is to be constructed from a square piece of cardboard, $6ft$ wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.
**Answer:** Problem taken directly from Fall’01 Exam 3 (look up the solution on the web)

9. Let $f(x)$ be a differentiable function such that $f(1) = f(2) = 0$. State a theorem and explain clearly why $f'(x)$ must have a root in the interval $(1,2)$. Does $f'(x)$ necessarily attain both positive and negative values in that interval?
**Answer:** State and apply the Mean Value Theorem (page 514 in your books). The answer of the question above is No (the only example however is $f(x) \equiv 0$!).