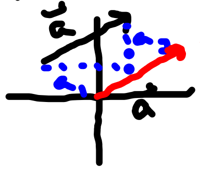
Definitions:

vector: a quantity with magnitude and direction

$\vec{a} = \langle a_1, a_2 \rangle$
horiz vert

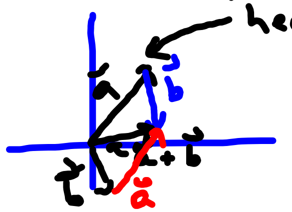


(a_1, a_2)
(Standard position)

1-1 correspondance between points and 2-D (standard) vectors

addition of vectors:

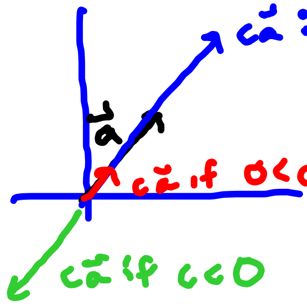
$$\langle a_1, a_2 \rangle + \langle b_1, b_2 \rangle = \langle a_1 + b_1, a_2 + b_2 \rangle$$



"head to tail"
or
Parallelogram

scalar multiplication of vectors:

$$c\langle a_1, a_2 \rangle = \langle ca_1, ca_2 \rangle$$



$c\vec{a}$ if $c > 1$

$c\vec{a}$ if $0 < c < 1$

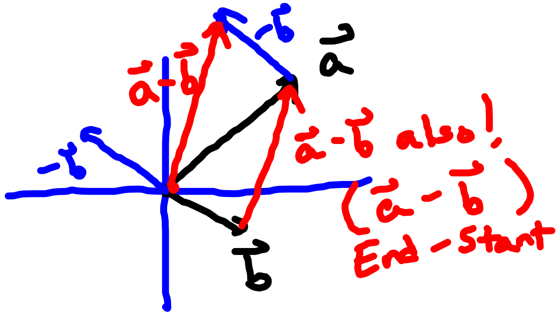
$c\vec{a}$ if $c < 0$

* parallel vectors are scalar multiples of each other.

subtraction of vectors:

$\langle a_1, a_2 \rangle - \langle b_1, b_2 \rangle$

$= \langle a_1 - b_1, a_2 - b_2 \rangle$



magnitude- If $\vec{a} = \langle a_1, a_2 \rangle$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2}$

unit vector- a vector whose magnitude is 1

i and j- $\vec{i} = \langle 1, 0 \rangle$

$\vec{j} = \langle 0, 1 \rangle$

Note $\langle a_1, a_2 \rangle = \langle a_1, 0 \rangle + \langle 0, a_2 \rangle$

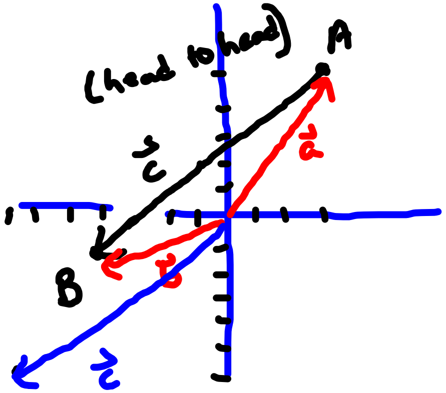
$= a_1 \langle 1, 0 \rangle + a_2 \langle 0, 1 \rangle$

$= a_1 \vec{i} + a_2 \vec{j}$ $\vec{i}/\vec{j}$ notation

Properties of Vectors: see p51

Examples:

Find a vector which represents the directed line segment from $A(3,5)$ to $B(-4,-1)$. Sketch $\overline{AB}$ and the vector in standard position.



$$\vec{c} = \vec{b} - \vec{a}$$
$$= (-4\vec{i} - 1\vec{j}) - (3\vec{i} + 5\vec{j})$$
$$= (-4-3)\vec{i} + (-1-5)\vec{j}$$
$$= -7\vec{i} - 6\vec{j}$$

$2\vec{i}+3\vec{j}$

Given a =< 2, 3 > and b = −i − 2j, find each of the following:
a + b, a − b. 2a. 3a + 4b, |a|, and a unit vector in the direction of b.

$$\vec{a}+\vec{b} = (2\vec{i}+3\vec{j}) + (-1\vec{i}-2\vec{j})$$
$$= (2+-1)\vec{i} + (3+^{-}2)\vec{j}$$
$$= \boxed{\vec{i}+\vec{j}}$$

$$\vec{a}-\vec{b} = (2-^{-}1)\vec{i} + (3-^{-}2)\vec{j}$$
$$= \boxed{3\vec{i}+5\vec{j}}$$

$$2\vec{a} = 2(2\vec{i}+3\vec{j}) = \boxed{4\vec{i}+6\vec{j}}$$

$$3\vec{a}+4\vec{b} = (6\vec{i}+9\vec{j}) + (-4\vec{i}-8\vec{j})$$
$$= \boxed{2\vec{i}+\vec{j}}$$

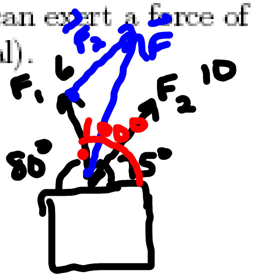
$$|\vec{a}| = \sqrt{2^2+3^2} = \boxed{\sqrt{13}}$$

$$\vec{b} = -\vec{i}-2\vec{j} \qquad |\vec{b}| = \sqrt{(-1)^2+(-2)^2} = \sqrt{5}$$

$$\vec{u}_{\vec{b}} = \frac{1}{\sqrt{5}}(-\vec{i}-2\vec{j}) = \boxed{\frac{-1}{\sqrt{5}}\vec{i} - \frac{2}{\sqrt{5}}\vec{j}}$$

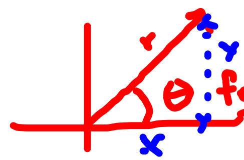
View from overhead

Two people are to pull ropes attached to a 50kg box (on a frictionless surface) as shown in the figure given in class. The person to the left of the box can exert a force of 6 N; the person on the right can exert a force of 10 N. Determine how fast the box accelerates and at what angle (from the vertical).



$\vec F = \vec F_1 + \vec F_2$ (resultant)

"Polar form"



θ from pos x-axis

$x = r\cos\theta \quad y = r\sin\theta$

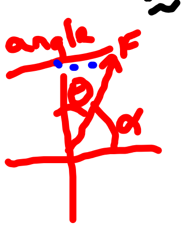
$\vec F_2 = (10\cos 75^\circ)\vec i + (10\sin 75^\circ)\vec j$

$\vec F_1 = (6\cos 100^\circ)\vec i + (6\sin 100^\circ)\vec j$

or $-6\cos 80^\circ$

$\vec F = (6\cos 100^\circ + 10\cos 75^\circ)\vec i + (6\sin 100^\circ + 10\sin 75^\circ)\vec j$

$\approx 1.55\,\vec i + 15.6\,\vec j$



$\tan\alpha = \frac{y}{x}$

$\alpha = \tan^{-1}\left(\frac{15.6}{1.55}\right) \approx 84.3^\circ$

$\theta = 90 - \alpha = \boxed{5.7^\circ}$

OR $\theta = \tan^{-1}\left(\frac{x}{y}\right)$

accel rate: $\vec F = m\vec a \qquad \vec a = \frac{1}{m}\vec F = \frac{1}{50}(1.55\vec i + 15.6\vec j)$

$= .031\vec i + .312\vec j$

$|\vec a| = \sqrt{(.031)^2 + (.312)^2} \approx \boxed{.313\ \frac{m}{s^2}}$