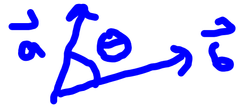
1.2-Dot Product



Definitions:

The *dot product* of the vectors a and b is given by $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Derivation of the Computational Formula:

(in text)

$\vdots$

if $\vec{a} = a_1\vec{i} + a_2\vec{j}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j}$,

then $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2$ <u>Scalar</u>

Properties of the Dot Product- in text $\vec{a}\cdot\vec{a} = |\vec{a}|^2$

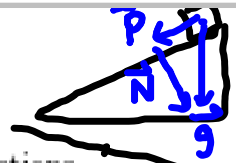
From the defintion of the dot product, it follows that the angle between 2 vectors is given by $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

a and b are *orthogonal* $\perp$ if and only if $\vec{a}\cdot\vec{b} = 0$

(2-D) only If $\vec{a} = a_1\vec{i} + a_2\vec{j}$, then $\vec{a}^{\perp} = {}^{-}a_2\vec{i} + a_1\vec{j}$

<u>Orthogonal Complement</u>

Intro:



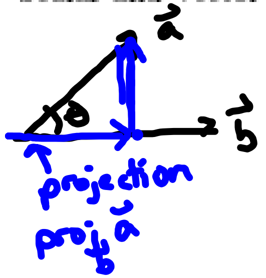
Block on Ramp

Resolve $\vec{g}$ into 2 components

(one // to ramp, 1 $\perp$ to ramp)

"projected $\vec{g}$ onto $\vec{p}$"

Scalar and Vector Projections-



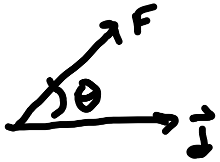
Project $\vec{a}$ onto $\vec{b}$:

$$|\text{proj}_{\vec{b}}\vec{a}| = |\vec{a}|\cos\theta$$
$$= \cancel{|\vec{a}|}\left(\frac{\vec{a}\cdot\vec{b}}{\cancel{|\vec{a}|}\,|\vec{b}|}\right)$$

$$\boxed{\text{comp}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}}$$ Scalar Projection

$$\boxed{\text{proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\,\frac{1}{|\vec{b}|}\cdot\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\,\vec{b}}$$ Vector Projection

Work- A force exerted over a displacement



$$W = \vec{F}\cdot\vec{d}$$
$$= |\vec{F}|\,|\vec{d}|\cos\theta$$

Examples:

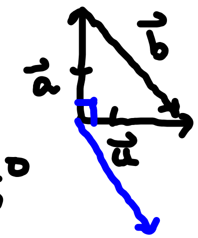
Find a · b: $\vec{i}-\vec{j}$ $\quad\quad$ $\vec{i}+2\vec{j}$

if a =< 1, −1 > and b =< 1, 2 >

$$\vec{a}\cdot\vec{b} = (1)(1) + (-1)(2)$$

$$= 1 - 2 = \boxed{-1}$$

in the figure given in class



$\vec{u}$ is a unit vector

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

$$= \underbrace{1}_{\text{unit}} \cdot \underbrace{\sqrt{2}}_{\text{Pyth}} \cdot \cos 135^{\circ}$$

$$= \sqrt{2}\cdot -\frac{\sqrt{2}}{2} = -\frac{2}{2} = \boxed{-1}$$

$\vec{i} + 2\vec{j}$

Find the angle between the vectors $< 1, 2 >$ and $6\mathrm{i} - 8\mathrm{j}$.

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(1)(6) + (2)(-8)}{\underbrace{\sqrt{1^2+2^2}}_{\sqrt{5}} \cdot \underbrace{\sqrt{6^2+(-8)^2}}_{10}} = \frac{-10}{10\sqrt{5}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)}$$

Find x such that $\mathrm{a} =< x, 1 > \perp \mathrm{b} =< 4 + x, 3 >$

Dot product: $\vec{a} \cdot \vec{b} = 0$

$$(x)(4+x) + (1)(3) = 0$$

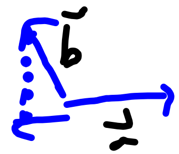
$$x^2 + 4x + 3 = 0$$

$$(x+3)(x+1) = 0$$

$$\boxed{x = -3, -1}$$

$4\vec{i}+5\vec{j} \qquad \vec{i}-2\vec{j}$

If a =< 4, 5 > and b =< 1, −2 >, find the scalar and vector projection of b onto a



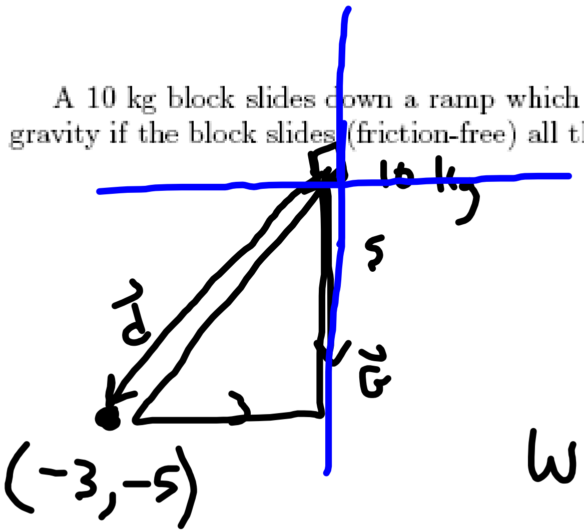
$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{4(1)+5(-2)}{\sqrt{4^2+5^2}} = \boxed{\frac{-6}{\sqrt{41}}}$$

$$\text{proj}_{\vec{a}}\vec{b} = \left(\frac{-6}{\sqrt{41}}\right)\frac{1}{|\vec{a}|}\vec{a}$$

$$= \frac{-6}{\sqrt{41}}\cdot\frac{1}{\sqrt{41}}(4\vec{i}+5\vec{j}) = \frac{-6}{41}(4\vec{i}+5\vec{j})$$

$$= \boxed{-\frac{24}{41}\vec{i} - \frac{30}{41}\vec{j}}$$

A 10 kg block slides down a ramp which is 5 m tall and 3 m horizontal. Find the work done by gravity if the block slides (friction-free) all the way down the ramp.



$$\vec{G} = -98\vec{j}$$

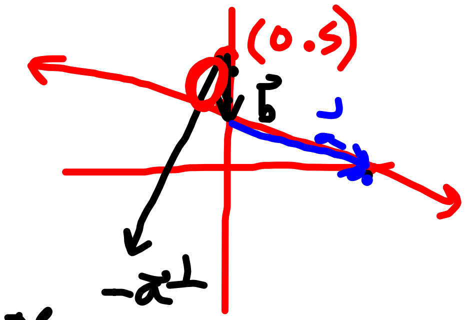
$$\vec{d} = -3\vec{i} - 5\vec{j}$$

$$W = \vec{G}\cdot\vec{d}$$

$$= (0)(-3) + (-98)(-5)$$

$$= \boxed{490 \text{ Nm or J}}$$

Find the distance from the point (0, 5) to the line $x + 2y = 6$



1) Find a vector from point to line

(0, 3) on line $\vec b = 3\vec j - 5\vec j = -2\vec j$

2) Find a vector in the direction of line

(0, 3) and (6, 0) $\vec a = 6\vec i - 3\vec j$

3) Find vector $\perp$ to line

$\vec a^{\perp} = 3\vec i + 6\vec j$ (or $-3\vec i - 6\vec j$)

4) Project $\vec b$ onto $\vec a^{\perp}$ (Scalar)

$$\text{comp}_{\vec a^{\perp}} \vec b = \frac{\vec a^{\perp} \cdot \vec b}{|\vec a^{\perp}|} = \frac{(3)(0) + 6(-2)}{\sqrt{3^2 + 6^2}} = \frac{-12}{\sqrt{45}}$$

$$\text{dist} = |\text{comp}_{\vec a^{\perp}} \vec b| = \frac{12}{\sqrt{45}} = \frac{12}{3\sqrt{5}} = \boxed{\frac{4}{\sqrt{5}}}$$