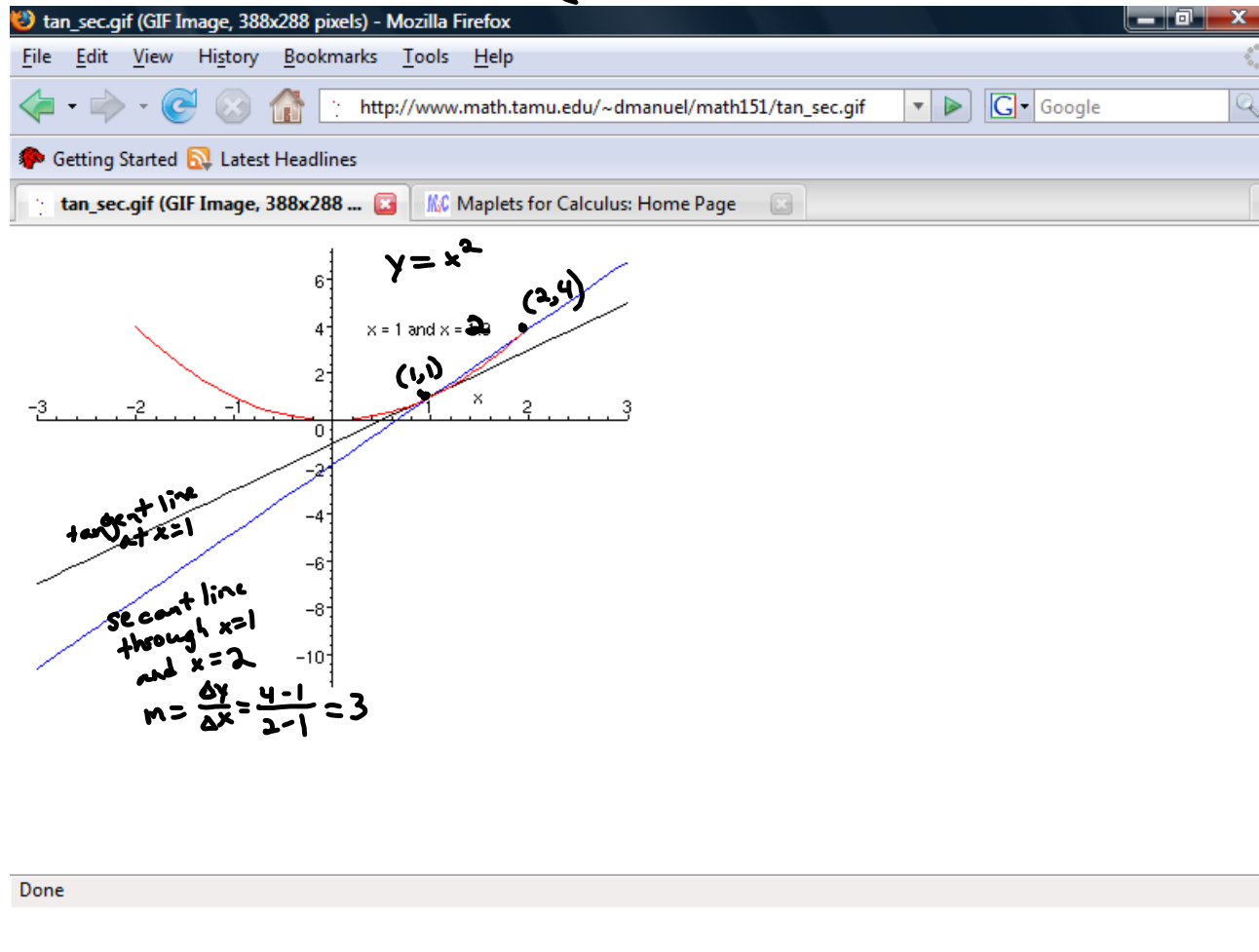


## 2.2: Numerical and Graphical Understanding of Limits

Motivation: The first historical problem of calculus

[http://www.math.tamu.edu/~dmanuel/math151/tan\\_sec.gif](http://www.math.tamu.edu/~dmanuel/math151/tan_sec.gif)

Find the slope of a line tangent to a curve  
(touches at one known point)



Recall: Notation of Limits:

$\lim_{x \rightarrow a^-} f(x) = L$  As  $x \rightarrow a$  from the left,  $f(x)$  (or  $y$ ) approaches  $L$   
( $x < a$ )  
left hand limit

$\lim_{x \rightarrow a^+} f(x) = L$  As  $x \rightarrow a$  from the right,  $f(x)$  or  $y$  approaches  $L$   
( $x > a$ )  
right hand limit

$\lim_{x \rightarrow a} f(x) = L$  As  $x \rightarrow a$  from both sides,  $f(x)$  or  $y$  approaches  $L$   
(2-sided limit) means  $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$

otherwise  
 $\lim_{x \rightarrow a} f(x)$  Does  
Not  
Exist

(we don't care what happens at  $x=a$ )

(for continuity, we do care, and  $L=f(a)$ )

# Maplet Example:

Maplets f Left and Right Limits and Continuity, using a Graph

File Edit

Getting tan\_sec

New Function Quit

Step 1 - Enter the limit from the left, the limit from the right and the value of the function in the boxes at the right.

NOTE: The one-sided limits and function value are integers.  
Notice the 3 numbers are independent, i.e. they may or may not be equal.

Step 2 - Decide if each statement is True or False.

$\lim_{x \rightarrow 5^-} f(x) =$

$\lim_{x \rightarrow 5^+} f(x) =$

$f(5) =$

$\lim_{x \rightarrow 5} f(x)$  exists.  T  F

$\lim_{x \rightarrow 5} f(x) = f(5)$   T  F

$f$  is continuous from the left.  T  F

$f$  is continuous from the right.  T  F

$f$  is continuous.  T  F

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