2.5-Continuity

**Definitions:** A function \( f \) is *continuous at \( x = a \) if and only if*

- \( f \) is continuous from the left at \( x = a \) if and only if
- \( f \) is continuous from the right at \( x = a \) if and only if
- \( f \) is continuous on the interval \([a, b]\) if and only if

**Definition:** \( f \) has a *removable discontinuity at \( x = a \) if and only if*

**Theorems:**

If \( \lim_{x \to a} g(x) = b \) and \( f \) is continuous at \( b \), then

Intermediate Value Theorem-

**Examples:**

Find the values of \( x \) for which \( f(x) = \frac{x^2 - 9}{x^2 - 5x + 6} \) is not continuous. Determine which, if any, of these discontinuities are removable.
Determine whether the function

\[ f(x) = \begin{cases} 
2x - 3 & \text{if } x \leq 2 \\
2x^2 & \text{if } x > 2
\end{cases} \]

is continuous at \( x = 2 \) or not and why. Sketch the graph of the function.

\[
\lim_{x \to 1} \left| \frac{x^2 + 2x - 3}{x^2 - 1} \right|
\]
Find the value of $k$ that makes

$$f(x) = \begin{cases} 
kx^2 & \text{if } x \leq 3 \\
2x + k & \text{if } x > 3
\end{cases}$$

continuous at $x = 3$.

Prove that there is at least one real solution to the equation $x^4 + x = 5$. Find an interval of length 1 which contains a solution.

**On Your Own:** 2.5 #1,2,12-18,29,31-34,37-39,42,43,47