2.5-Continuity

Definitions: A function f is *continuous* at $x = a$ if and only if

f is continuous from the left at $x = a$ if and only if

f is continuous from the right at $x = a$ if and only if

f is continuous on the interval $[a, b]$ if and only if

Definition: f has a *removable discontinuity* at $x = a$ if and only if

Theorems:

If $\lim_{x \to a} g(x) = b$ and f is continuous at b, then

Intermediate Value Theorem-

Examples:

Find the values of x for which $f(x) = \dfrac{x^2 - 9}{x^2 - 5x + 6}$ is not continuous. Determine which, if any, of these discontinuities are removeable.

Determine whether the function

$$f(x) = \begin{cases} 2x - 3 & \text{if } x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$$

is continuous at $x = 2$ or not and why. Sketch the graph of the function.

Compute $\lim_{x \to 1} \left| \frac{x^2 + 2x - 3}{x^2 - 1} \right|$

Find the value of k that makes

$$f(x) = \begin{cases} kx^2 & \text{if } x \le 3 \\ 2x + k & \text{if } x > 3 \end{cases}$$

continuous at $x = 3$.

Prove that there is at least one real solution to the equation $x^4 + x = 5$. Find an interval of length 1 which contains a solution.

On Your Own: 2.5 #1,2,12-18,29,31-34,37-39,42,43,47